Trade and Inequality: A Sufficient Statistics Approach

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Abstract

We develop a sufficient statistics approach for measuring the effects of international trade on within-country income inequality. We show that, when changes in within-sector inequality are only generated via linear profit sharing between individuals and firms, observing changes in two statistics - bilateral international trade flows and the share of exporters - is sufficient for calculating trade-induced changes in inequality. This holds in models with heterogeneous firms and monopolistic competition in Arkolakis et al. (2012) (ACR). Our approach complements the ACR formula, requiring only minimal additional data and allowing one to calculate the effects of trade on various inequality measures.

Keywords: Trade, Inequality, Gains, Gini, Sufficient Statistics

JEL-codes: F10; F11; F16.

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Introduction

Within the economics profession, it is widely accepted that most models of international trade generate positive aggregate gains from trade. In fact, Arkolakis et al. (2012) demonstrate that in a broad class of those models the aggregate gains from trade are a simple function of the expenditure share on domestic goods and the trade-elasticity parameter. This relationship has become known colloquially as the ACR formula.

In this paper, we explore what is perhaps the second most widely accepted conventional wisdom in economics about trade: that increases in trade lead to increases in within-country income inequality. We demonstrate that, for a subset of ACR formula-consistent trade models featuring heterogeneous firms and monopolistic competition as in Melitz (2003), the ability to observe changes in bilateral trade flows and the share of exporters is sufficient for calculating trade-induced changes in income inequality. Hence, we can infer changes in inequality without explicit modeling of inequality mechanisms.

The complementarity of our results with the ACR formula offers a simple way of calculating the effects of trade on inequality vis-à-vis aggregate welfare gains in a unified framework. First, we show that the ACR framework allows to specify the distribution of profits as a function of trade flows and bilateral share of exporters. Second, we require one additional restriction relative to the ones necessary for the ACR formula to hold, i.e., that income inequality is generated via a linear profit-sharing mechanism between firms and individuals. This allows us to map the distribution of firms' profits to income distribution using sufficient statistics. We show that the linear profit-sharing mechanism can be derived in frameworks based on various labor-market frictions, e.g., quasi-rent sharing, fair wages, search and matching, and others.

Our approach allows to derive analytical expressions for both the Lorenz curve and the Gini coefficient, and the total income shares held by various population groups (e.g., the top 5 percent); these can be derived as a function of (i) bilateral trade flows, (ii) the bilateral share of exporters, and (iii) the set of three parameters that capture the elasticity of substitution, dispersion of productivity across firms, and the share of profits accruing to individuals as incomes. Hence, conditional on the values of these three parameters, our results provide a self-sufficient framework for quantifying changes in within-sector inequality from arbitrary changes in trade without explicitly constructing or calibrating a full-fledged model with trade and inequality. We can either directly observe the intensive and extensive margins of trade before and after the shock, or we can calculate these margins by using a standard heterogeneous firms model (based on Melitz, 2003) that does not incorporate inequality but does adhere to the ACR formula restrictions.

We demonstrate the advantages of the proposed sufficient statistics approach by conducting three counterfactual experiments based on the data from 43 economies and the Rest of the World in 2000-2014. We are able to answer the following questions: How would inequality change if countries reverted to autarky? How large were trade-induced changes in inequality measures between 2000 and 2014? What is the extent of heterogeneity in the welfare gains from trade across individuals within countries? Our approach requires minimal data and provides quantitative answers in a framework where the ACR formula holds.

This paper relates to the literature on trade and inequality that relies on the heterogeneous-firms model in Melitz (2003). In this literature, trade-induced income inequality arises due to various labor-market frictions that connect individuals' incomes to the profit of the firm that employs them. Labor-market frictions can come in the form of fair wages (see Egger and Kreickemeier, 2009, 2012; Amiti and Davis, 2012; Egger et al., 2013), Mortenson-Pissarides search-and-matching frictions (see Helpman and Itskhoki, 2010; Helpman et al., 2010; Felbermayr et al., 2011; Antras et al., 2017), efficiency wages (see Davis and Harrigan, 2011), and other mechanisms that lead to positive assortative matching between high-profit firms and high-income individuals (among others see Manasse and Turrini, 2001; Yeaple, 2005; Sampson, 2014; Egger et al., 2021; Jha and Rodriguez-Lopez, 2021; Nigai, 2023). While acknowledging the intricacies of the exact mechanisms behind income inequalities, we approximate the link between firm profits and individuals' incomes by using a simple, linear profit-sharing mechanism in the spirit of the quasi-rent-sharing literature (see Card et al., 2018). However, we also show that our specification can be microfounded using alternative mechanisms, which allows us to quantify the effects of trade on inequality in an isomorphic way that does not depend on a particular model in mind.

We also relate to the broader literature on globalization and inequality. However, our focus is on changes in within-sector inequality exclusively generated by firm-level differences and linear profit sharing. Hence, our approach does not apply to works that consider Stolper-Samuelson-type effects, which may lead to wage inequality across individuals (Autor et al., 2014; Adao et al., 2022), skills (Parro, 2013; Burstein and Vogel, 2017), firms (Harrigan and Reshef, 2015), and sectors (Kim and Vogel, 2021; Rodrik, 2021). Our work also relates to recent papers that quantify the effects of sectoral trade shocks on labor market outcomes using micro data (see Hummels et al., 2014; Dhyne et al., 2022). We also relate to Galle et al. (2022), who extend the ACR formula to calculate group-specific wage effects of trade using sufficient statistics in Ricardian models. Relative to their work, our sufficient statistics approach can be applied in a different class of models based on heterogeneous firms where within-sector inequality is generated due to firm-selection effects.

This paper also relates to the literature that characterizes trade outcomes using sufficient statistics approaches, which dates back to Jones (1965). In this literature, observing trade flows in the initial equilibrium is sufficient for calculating various counterfactual trade outcomes so long as one knows a few key parameters (see, for example, Dekle et al., 2007; Caliendo and Parro, 2014; Costinot and Rodriguez-Clare, 2014). This literature includes research on calculating welfare gains from trade under various preference and market structures (see Arkolakis et al., 2012, 2019), and on analyzing the effects of trade policy (see Lashkaripour, 2021) and geographical barriers (see Anderson et al., 2018). We contribute by proposing a sufficient statistics approach for calculating the effects on inequality, while also preserving tractable features of the ACR formula. This allows us to calculate the effects of globalization on both aggregate welfare and inequality. Hence, our approach is related to Antras et al. (2017), Artuc et al. (2019), Adao et al. (2020), and Galle et al. (2022). Adao et al. (2020) extend the ACR formula by proposing a sufficient statistics approach that takes account of the impact of firm heterogeneity. We, on the other hand, develop an approach to calculate the impact of firm heterogeneity on distributional outcomes. We also relate to Nigai (2023), who considers the trade-off between aggregate gains and inequality in a Melitz-type framework. Our work is distinct and its advantages include consistency with the ACR formula, sufficient-statistics approach with minimal data requirements, and generalization of different income inequality mechanisms in an isomorphic linear profit-sharing rule between individuals and firms.

1 ACR-consistent model of trade with inequality

In this section, we specify a parsimonious model of international trade with inequality that is consistent with the ACR formula. We emphasize the fact that introduction of income inequality will not affect the two sufficient statistics required for the ACR formula to work: (i) domestic expenditure share and (ii) trade-elasticity parameter. This means that under our approach, calculating counterfactual effects of trade on inequality can be done using standard models without bringing the inequality mechanism out of the shadows.

We consider a subclass of models based on heterogeneous firms and monopolistic competition in Arkolakis et al. (2012) that adhere to the following restrictions:

(R1) Trade is balanced.

(R2) Aggregate profits are a constant share of revenues.

(R3) The import demand system is CES.

Let us now describe a framework in which restrictions **R1** through **R3** hold. There are J countries in the world. Each country $i \in J$ has L_i individuals and each individual has an innate entrepreneurial ability ϕ drawn from a known country-specific Pareto distribution with the cumulative distribution function $F_i(\phi) = 1 - b_i^{\theta}\phi^{-\theta}$, where $b_i > 0$ is a country-specific scale parameter and $\theta > 0$ is the shape parameter common to all countries. Individuals are randomly matched with ex-ante identical firms. Upon matching, entrepreneurial ability directly translates into the corresponding firm's productivity also denoted by ϕ.

Individuals have different roles in the economy and act as: (i) potential entrepreneurs, (ii) workers, (iii) consumers, and (iv) shareholders. Because we are interested in income inequality, we index individuals and firms that are matched with them using their positions in the income and productivity distributions denoted by $\rho \in [0, 1]$. Let us also use $y_i(\rho)$ to denote the total nominal income of consumer ρ. By construction, ρ follows a uniform distribution with support on [0,1]. Consumers maximize the following CES utility function subject to the budget constraint:

$$U_j = \left(\sum_{i \in J} \int_{\mathbb{R}_{ij}} q_{ij}(\rho)^{\frac{\sigma-1}{\sigma}} d\rho \right)^{\frac{\sigma}{\sigma-1}},$$

where σ is the parameter governing the elasticity of substitution between varieties, and $\mathbb{R}_{ij} \subseteq [0, 1]$ is the set of all firms from i that sell goods in j. The aggregate demand for variety ρ can then be specified as:

$$q_{ij}(\rho) = p_{ij}(\rho)^{-\sigma} P_j^{\sigma-1} L_j y_j, \text{ where } y_j = \int_0^1 y_j(\rho) d\rho.$$

When productivity distribution is Pareto, the quantile that corresponds to the position ρ is equal to $b_i(1-\rho)^{-\frac{1}{\theta}}$. Let w_i and τ_{ij} denote the cost of an input bundle and variable trade costs such that we can specify the total marginal cost of firm ρ from country i that serves market j as:

$$m_{ij}(\rho) = \underbrace{(1-\rho)^{\frac{1}{\theta}}}_{\text{Firm component}} \cdot \underbrace{\left(\frac{\tau_{ij} w_i}{b_i}\right)}_{\text{Country-pair component}} .$$

Here b_i acts as a country shifter that governs the country-specific component of the marginal cost, and $(1-\rho)^{\frac{1}{\theta}}$ is a firm-specific component that is decreasing in ρ such that more productive firms have lower marginal costs. The profit function of firm ρ from selling goods in market j can then be specified as:

$$\pi_{ij}(\rho) = p_{ij}(\rho)^{1-\sigma} P_j^{\sigma-1} L_j y_j - m_{ij}(\rho) p_{ij}(\rho)^{-\sigma} P_j^{\sigma-1} L_j y_j - w_j f_{ij},$$

where f_{ij} denotes the fixed cost of export. Taking the first-order condition with respect to $p_{ij}(\rho)$ leads to the familiar constant markup pricing rule:

$$p_{ij}(\rho) = \frac{\sigma}{\sigma-1} m_{ij}(\rho).$$

The marginal firm that serves market j can then be specified using the zero-profit condition:

$$m_{ij}(\rho^*) = \frac{\sigma-1}{\sigma}\left(\frac{\sigma w_j f_{ij}}{L_j y_j}\right)^{\frac{1}{1-\sigma}} P_j \text{ and } \rho_{ij}^* = m_{ij}^{-1}(\rho^*)$$

Total profits accruing to firm ρ in all markets can then be written as:

$$\Pi_i(\rho) = \sum_j \pi_{ij}(\rho) = \sum_j \mathbb{1}_{\rho>\rho_{ij}^*}\left\{\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1} m_{ij}(\rho)\right)^{1-\sigma} P_j^{\sigma-1} L_j y_j\right\},$$

where $\mathbb{1}_{\rho>\rho_{ij}^*}$ is an indicator function that takes the value of one whenever $\rho > \rho_{ij}^*$ and takes the value of zero otherwise.

As we mention above, once an entrepreneur ρ is matched with a firm, her ability, $b_i(1-\rho)^{-\frac{1}{\theta}}$, directly translates into firm productivity parameter. We follow the literature on "quasi-rent" sharing as in Card et al. (2014) and Card et al. (2018) to characterize how total profits are shared between entrepreneurs and firms. However, we show in Section 3 that the results are robust to using multiple alternative microfoundations of the profit-sharing mechanism.

A firm and an entrepreneur bargain over total profit $\Pi_i(\rho)$. Let $\widetilde{y}_i(\rho)$ denote total profits that accrue to the entrepreneur such that the firm retains the remaining $\widetilde{\Pi}_i(\rho) = \Pi_i(\rho) - \widetilde{y}_i(\rho)$. The firm and the entrepreneur bargain over the total payoff via a cooperative game maximizing the following Nash product:

$$O_i(\rho) = [\widetilde{y}_i(\rho) - y_i^0]^\alpha [\widetilde{\Pi}_i(\rho)]^{1-\alpha}.$$

where $\alpha \in [0, 1]$ measures the relative bargaining power of the entrepreneur and y_i^0 denotes her outside option. If the bargaining fails, the firm shuts down and earns zero profit and the entrepreneur earns the value of an outside option. As a solution to the Nash bargaining problem, the optimal transfer is obtained as follows:

$$\widetilde{y}_i(\rho) = (1 - \alpha) y_i^0 + \alpha \Pi_i(\phi),$$

where the share of total profits accruing to the entrepreneur is proportional to her bargaining power. For simplicity, we assume that the value of an entrepreneur's outside option is zero such that she simply receives a fixed share $\alpha \in [0, 1]$ of total profits.[1] In essence, this is equivalent to assuming that before the individual ρ is matched with a firm, she writes an enforceable contract such that she keeps share α of firm's ex-post total profits.[2] Remaining profits $(1 - \alpha)\Pi_i(\rho)$ are distributed among shareholders as dividends. In the baseline case, we assume that shareholders hold equal shares of each firm but relax this assumption in an extension discuss in Section 5.4.

In addition to the potential entrepreneurial income and dividends, each individual also collects wage from supplying labor. We assume that each individual has one unit of labor endowment such that L_i reflects the number of individuals as well total labor endowment in country i. Individuals supply labor inelastically at wage w_i, independently of whether they collect entrepreneurial income. Hence, entrepreneurial income is interpreted as a premium tied to profits that ρ receives on top of the average wage w_i.

[1]Alternatively, it is possible to specify a framework where the bargaining between entrepreneurs and firms is subject to the holdup problem due to the matching costs. Let $w_i f_i^e$ denote ex-ante matching cost that the firm must pay to be matched with individual ρ. If the bargaining is not successful the firm can recover the share δ of this cost. Then, total profit becomes a function of total revenues $R_i(\phi)$ net of $w_i f_i^e$. The bargaining then maximizes the following generalized Nash objective $O_i(\rho) = [\widetilde{y}_i(\rho) - y_i^0]^\gamma [\widetilde{\Pi}_i(\rho) + (1-\delta) w_i f_i^e]^{1-\gamma}$. Following Card et al. (2018), we can derive income that accrues to the entrepreneur as $\widetilde{y}_i(\phi) = (1-\gamma) y_i^0 + \gamma(1-\delta) w_i f_i^e + \gamma \Pi_i(\phi)$. Note that in this case $\widetilde{y}_i(\phi)$ is still a linear function of $\Pi_i(\phi)$. Hence, our results would remain robust under this alternative specification.

[2]While we do not allow α to vary across countries, our approach would also remain valid if this parameter was country-specific.

Total nominal income of the individual associated with firm ρ can then be specified as:

$$y_i(\rho) = \underbrace{w_i}_{(i)} + \underbrace{\alpha\Pi_i(\rho)}_{(ii)} + \underbrace{d_i}_{(iii)}, \text{ where } d_i = (1-\alpha)\int_\rho \Pi_i(\rho)d\rho. \tag{1}$$

Three income components in equation (1) are interpreted as follows: (i) the wage that ρ receives for her labor endowment, (ii) the share of profits of the firm as entrepreneurial income, and (iii) the dividends received from other firms if ρ owns their shares. The income equation is also consistent with Nigai (2023). However, relative to his work, we are able to study both welfare and inequality using a novel sufficient statistics approach as well as provide multiple alternative microfoundations for the expression for nominal income.

Since there are L_i potential entrepreneurs in country i, the potential number of entrant firms is also given by L_i, consistent with Chaney (2008). As dividends are distributed to L_i individuals, the term d_i is simply $(1-\alpha)$ share of average profits.

This brings us to the fourth restriction necessary for the sufficient statistics approach to hold:

(R4) The profit-sharing mechanism is linear such that individuals keep α share of firms' total profits.

Hence, relative to Arkolakis et al. (2012), we require one additional assumption **R4** to study the effects of trade on inequality. Taking expectation in equation (1) allows us to specify average income as:[3]

$$y_i = \frac{\sigma\theta}{\sigma\theta - (\sigma-1)} w_i. \tag{2}$$

As we show in Appendix A, the trade flows from i to j, X_{ij}, and trade shares, λ_{ij} can be derived as follows:

$$X_{ij} = \lambda_{ij} L_j y_j, \text{ where } \lambda_{ij} = \frac{L_i b_i^\theta (w_i \tau_{ij})^{-\theta} f_{ij}^{1-\frac{\theta}{\sigma-1}}}{\sum_k L_k b_k^\theta (w_k \tau_{kj})^{-\theta} f_{kj}^{1-\frac{\theta}{\sigma-1}}}. \tag{3}$$

[3]As usual, we assume that $\theta > \sigma - 1$.

We can also express the CES price index as follows:

$$P_j = A_j \left(\sum_k L_k b_k^\theta (w_k \tau_{kj})^{-\theta} f_{kj}^{1-\frac{\theta}{\sigma-1}} \right)^{-\frac{1}{\theta}}, \tag{4}$$

where A_j is a j-specific constant.[4] Finally, to close the model we turn to the trade balance condition and use the relationship in equation (2) to specify the condition that solves for wages as:

$$L_i w_i = \sum_j \lambda_{ij} L_j w_j. \tag{5}$$

Hence, equilibrium is characterized by the vector of wages and prices $\{w_i, P_i\}$ such that conditions in equations (3), (4), and (5) are satisfied. Note that once the expression for y_i in equation (3) is replaced with the relationship in equation (2), calculating the equilibrium does not require information on income distribution. Hence, income inequality does not matter for equilibrium trade flows or average welfare, which implies that the sufficient statistics approach developed in this paper preserves the advantages of the ACR formula and can be applied without explicitly modeling inequality.

2 Sufficient statistics

In this section, we develop a sufficient statistics approach to quantifying the effects of trade on inequality. Before that, let us restate the ACR formula using the expressions derived in Section 1. Combining the expression for internal trade shares in equation (3) and the CES price index in equation (4) allows us to express real wages in country j as:

$$\omega_j = \frac{w_j}{P_j} = B_j \lambda_{jj}^{-\frac{1}{\theta}}, \tag{6}$$

where B_j is a country-specific constant. Let a be an arbitrary variable, and let a' denote its counterfactual value such that the relative change is defined as $\widehat{a} = a'/a$. Then, according to equation (6), the ACR formula states that the relative change in real wage is a function of only two sufficient statistics: $\widehat{\lambda}_{jj}$ and θ. As we show in equation (2), average income, y_i, is proportional to wage, w_i, up to a constant. Hence, the formula in equation (6) can also be used to calculate changes in the average welfare.

[4] We define $A_j = \left(\frac{\theta}{\theta-(\sigma-1)} \left(\frac{\sigma}{\sigma-1} \right)^{-\theta} \left(\frac{\theta}{\sigma\theta-(\sigma-1)} \right)^{\frac{\theta}{\sigma-1}-1} L_j^{\frac{\theta-(\sigma-1)}{\sigma-1}} \right)^{-\frac{1}{\theta}}$.

Next, we derive sufficient statistics for three inequality outcomes: (i) the income of individual ρ relative to the average income; (ii) the most widely used measure of income inequality, the Gini coefficient; and (iii) the share of total income held by the top δ share of the population.

We show that the sufficient statistics for calculating outcomes (i)-(iii) are trade flows X_{ij} (equivalently λ_{ij} and $L_j y_j$), the share of exporter firms ρ^*_{ij}, and parameters $\{\sigma, \theta, \alpha\}$. We start with simplifying the expression for dividends in equation (1):

$$d_i = (1-\alpha)\frac{\sigma-1}{\sigma\theta}\frac{1}{L_i}\sum_j X_{ij}.$$

Together with the identity in equation (2), we derive the following Lemma 1.

Lemma 1 *The nominal income of individual ρ is given as:*

$$y_i(\rho) = y_i\left(1 - \frac{\alpha(\sigma-1)}{\sigma\theta}\right) + \alpha\Pi_i(\rho), \tag{7}$$

and the profits of firm ρ can be expressed as:

$$\Pi_i(\rho) = \sum_j \mathbb{1}_{\rho>\rho^*_{ij}}\left\{\frac{X_{ij}}{L_i}\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{1}{(1-\rho^*_{ij})}\left[\left(\frac{1-\rho}{1-\rho^*_{ij}}\right)^{\frac{1-\sigma}{\theta}} - 1\right]\right\}.$$

Proof. See Appendix A. ■

In equation (7), the nominal income, $y_i(\rho)$, consists of country-specific wage and dividends, expressed as the first term common to all, and ρ-specific entrepreneurial income as a share of profits in the second term. It immediately follows from Lemma 1 that the nominal income of ρ relative to the average income, $r_i(\rho) = y_i(\rho)/y_i$, can be specified as:

$$r_i(\rho) = \left(1 - \alpha\frac{\sigma-1}{\sigma\theta}\right) + \alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\sum_j \mathbb{1}_{\rho>\rho^*_{ij}}\left\{\lambda_{ij}\frac{L_j y_j}{L_i y_i}\frac{1}{(1-\rho^*_{ij})}\left[\left(\frac{1-\rho}{1-\rho^*_{ij}}\right)^{\frac{1-\sigma}{\theta}} - 1\right]\right\}, \tag{8}$$

where the first term captures relative income from wages and dividends and the second term expresses relative entrepreneurial income. The latter highlights that profits depend on the size of trade volume, $\lambda_{ij}\frac{L_j y_j}{L_i y_i}$, and the distance of productivity ρ relative to the cutoff ρ^*_{ij}. Intuitively, a firm with $\rho = \rho^*_{ij}$ earns zero profit in market j, while its profit would be higher with a higher level of ρ. Parameter α plays a central role in determining the total income of ρ relative to the average income. When $\alpha = 0$, the model collapses to the standard Melitz framework with perfect equality as $r_i(\rho) = 1$ for all ρ. However, as α increases the second term in equation (8) becomes more important. Hence, at higher values of α, the effect of trade on relative income differences is higher.

Note that to calculate measures of inequality, one only needs to know relative incomes $r_i(\rho)$. Hence, our approach in equation (8) suggests that it is sufficient to observe X_{ij}, ρ^*_{ij} as well as set values for $\{\sigma, \theta, \alpha\}$. Next, to develop intuition on how these observable statistics and structural parameters drive differences across income percentiles, let us consider an example of two percentiles ρ and $\tilde{\rho}$ such that $\tilde{\rho} > \rho$.

Conditional on ρ and $\tilde{\rho}$ exporting to the same set of markets, the difference between $r_i(\rho)$ and $r_i(\tilde{\rho})$ is governed by parameters $\{\sigma, \theta\}$ such that $\theta > \sigma - 1$. On the one hand, the income difference is increasing as σ gets closer to $\theta + 1$ as goods become more substitutable and more productive firms get relatively higher profits. On the other hand, the difference is decreasing in θ because higher θ reduces the dispersion of productivities and profits across firms. Next, suppose that $\tilde{\rho}$ exports to an additional market j relative to ρ. In this case, the differences between $r_i(\rho)$ and $r_i(\tilde{\rho})$ is also amplified by the term $\lambda_{ij} \frac{L_j y_j}{L_i y_i}$. Intuitively, the sufficient statistic captures the access to market j in terms of the trade share λ_{ij} as well as its importance in terms of its relative size $\frac{L_j y_j}{L_i y_i}$.

Under an arbitrary counterfactual shock to trade costs, one can recover $X_{ij}{}'$ and $\rho^*_{ij}{}'$ and consequently $r_i(\rho)'$. This means that the welfare changes for individual ρ can be calculated as:

$$\widehat{\omega}_j(\rho) = \widehat{r}_j(\rho) \cdot \widehat{\lambda}_{jj}^{-\frac{1}{\theta}}. \tag{9}$$

The expression for relative incomes in equation (8) allows us to analytically characterize the Lorenz curve and the Gini coefficient as functions of observable sufficient statistics.[5] Since the Lorenz curve measures the share of cumulative income captured by individuals below each percentile ρ, we can obtain the expression for it by integrating the relative income in equation (8).

Proposition 1 *The expression for the Lorenz curve is represented by:*

[5]Note that the income ranking across individuals, ρ, is stable across different equilibria, which allows us to formulate changes in welfare as a function of ρ. This is because in Melitz-type models, profits are monotonically increasing in firm productivity, and since incomes are directly linked to profits, the income ranking is stable.

$$\mathcal{L}_i(\rho) = \left(1 - \alpha\frac{\sigma-1}{\sigma\theta}\right)\rho + \alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\sum_j \mathbb{1}_{\rho>\rho^*_{ij}}\Lambda_{ij}(\rho), \tag{10}$$

where:

$$\Lambda_{ij}(\rho) = \lambda_{ij}\frac{L_j y_j}{L_i y_i}\left(\frac{\theta}{\theta-(\sigma-1)}\left[1 - \left(\frac{1-\rho'}{1-\rho^*_{ij}}\right)^{\frac{1-\sigma}{\theta}+1}\right] - 1 + \left(\frac{1-\rho}{1-\rho^*_{ij}}\right)\right), \tag{11}$$

and the expression for the Gini coefficient is given by:

$$\mathcal{G}_i = \alpha\frac{\sigma-1}{\sigma\theta} - \alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{2\theta-(\sigma-1)}\left(\sum_j \lambda_{ij}(1-\rho^*_{ij})\frac{L_j y_j}{L_i y_i}\right). \tag{12}$$

Proof. See Appendix A. ■

The analytical expression for the Lorenz curve is valuable because it allows us to characterize both the shares of income of different income groups and the Gini coefficient. Next, we use the expressions for the Lorenz curve and the Gini coefficient to characterize relative changes in different measures of inequality.

Proposition 2 *The relative change in the share of total income that accrues to the top $100 \cdot \delta$ percent of the population can be calculated as follows:*

$$\widehat{\mathcal{S}}_i(\delta) = \frac{1-\mathcal{L}'_i(1-\delta)}{1-\mathcal{L}_i(1-\delta)} \ \textit{for} \ \delta > 0, \tag{13}$$

and the relative change in the Gini coefficient can then be calculated as:

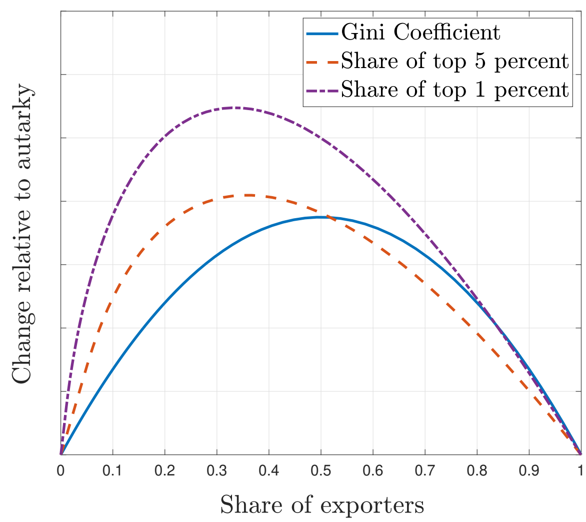
$$\widehat{\mathcal{G}}_i = \frac{\mathcal{G}'_i}{\mathcal{G}_i} \ \textit{for} \ \alpha > 0, \tag{14}$$

where $\mathcal{L}'_i(1-\delta)$ and $\mathcal{G}'_i$ are functions of ${X_{ij}}'$ and ${\rho^*_{ij}}'$ observed in a counterfactual equilibrium.

Proof. The expressions follow from the definition of the counterfactual change, $\widehat{a} = a'/a$. ■

Note that calculating welfare changes for individual ρ and changes in the income share of top $100 \cdot \delta$ percent, $\{\widehat{\omega}_j(\rho), \widehat{\mathcal{S}}_i(\delta)\}$, only requires observations of trade flows and bilateral share of exporters in the initial and counterfactual equilibria, and knowledge of three parameters $\{\sigma, \theta, \alpha\}$. Moreover, as equations (12) and (14) suggest, we do not require the value of α to calculate the relative change in the Gini coefficient, $\widehat{\mathcal{G}}_i$. The reason for this is that the level of the Gini coefficient increases proportionately with α according to equation (12). Intuitively, a larger value of α means that entrepreneurs keep a larger share of firms' profits, which leads to larger income differences across them and higher Gini coefficient. Conveniently, this linear relationship between α and $\mathcal{G}_i$ also implies that as long as $\alpha > 0$, we can characterize relative changes in the Gini coefficient without explicitly specifying the value of this parameter as it cancels out when calculating relative changes.

At this point, it is instructive to consider changes in trade-induced inequality measures relative to autarky. For that, we simulate the world economy consisting of I symmetric countries. We plot the Gini coefficient and the income shares of the top 1 percent and the top 5 percent of the population in Figure 1.



Notes: The figure plots relative changes in the Gini coefficient and the income shares of the top 5 and the top 1 percent of the population against the share of exporters based on a simulated world economy with $I = 100$ symmetric countries, where $\rho^*_{ii} = 0$. The changes (vertical axis) are calculated using the share of exporters (horizontal axis) and trade flows in hypothetical equilibria relative to autarky.

Figure 1: Inequality Measures vs. Share of Exporters

We plot $\widehat{\mathcal{G}}_i$, $\widehat{\mathcal{S}}_i(0.05)$, and $\widehat{\mathcal{S}}_i(0.01)$ against the share of exporters. We note that the responses of all three inequality measures are concave. The share of the top 1% income peaks first as trade benefits only the most productive firms when the share of exporters is low. The share of the top 5% and the Gini coefficient display similar patterns but reach their maxima at higher levels of the share of exporters. Initially, increasing the number of exporters also increases inequality because the share of exporting firms is low at that point, and the effects are limited to only a few individuals in the right tail of the income distribution. As more firms become exporters, however, the effects taper off, and eventually, inequality starts to decrease converging to the autarky level when all firms are exporters. Indeed, inspecting equation (12), assuming autarky ($\lambda_{ii} = 1$) gives the following expression for the Gini coefficient:

$$\mathcal{G}_i^{autarky} = \alpha\frac{\sigma-1}{\sigma\theta} - 2\alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{4\theta-2(\sigma-1)}(1-\rho_{ii}^*).$$

At the other extreme, we can examine the case of selectionless trade (all firms export everywhere such that $\rho_{ij}^* = 0$ for all ij). We follow Eaton and Kortum (2002) and refer to this case as zero-gravity trade:

$$\mathcal{G}_i^{zero-gravity} = \alpha\frac{\sigma-1}{\sigma\theta} - 2\alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{4\theta-2(\sigma-1)}\left(1-\rho_{ii}^*\lambda_{ii}\right).$$

Note that the expression for $\mathcal{G}_i^{autarky}$ and $\mathcal{G}_i^{zero-gravity}$ can be further simplified if we assume no barriers to domestic entry, i.e, $\rho_{ii}^* = 0$. In that case, the following holds:

$$\overline{\mathcal{G}}_i = \mathcal{G}_i^{autarky} = \mathcal{G}_i^{zero-gravity} = \alpha\frac{\sigma-1}{\sigma\theta} - 2\alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{4\theta-2(\sigma-1)}.$$

The qualitative pattern that inequality is concave between the two extreme cases of autarky and zero-gravity trade is consistent with the results in the literature that have also found an inverted U-shaped relationship between the share of exporters and aggregate measures of inequality (for example, see Helpman et al., 2010; Egger et al., 2013). In this respect, the approach in this paper is isomorphic to several leading models of trade and inequality. In addition, its analytical tractability allows us to characterize two of the most common measures of inequality using sufficient trade statistics.

In terms of the real-world applications, the results in Figure 1 for $1-\rho_{ij}^* > 0.5$ are unlikely to be useful as the share of exporters on a bilateral level above 50% is rarely observed in the data. Ultimately, the data on X_{ij} and ρ_{ij}^* will determine the relevant part of the support. Fortunately, more data on ρ_{ij}^* are becoming available, and we illustrate the usefulness of our approach in data applications in Section 4.

3 Alternative Microfoundations of Linear Profit Shifting

In this section, we discuss multiple alternative ways to microfound the linear profit sharing in **R4**. Due to the sheer volume of the literature on trade and inequality, fitting multiple existing inequality frameworks under a single umbrella requires us to preserve the income structure of the baseline model, i.e, individuals have three sources of income: (i) homogeneous wage for labor endowment, (ii) part of total profits, and (iii) potential dividends. This means that total nominal income can still be described as follows:

$$y_i(\rho) = w_i + \alpha \Pi_i(\rho) + d_i. \tag{15}$$

Given this structure, we can microfound α and linear profit sharing using various existing frameworks. In some cases, we do not need to impose additional functional form and parameter restrictions. However, such restrictions are elemental for some existing frameworks to produce the linear-profit-sharing rule as an equilibrium outcome.

The advantage of relating our baseline model to a menu of feasible microfoundations is that it allows researchers to utilize our sufficient statistics results without adopting a specific mechanism for inequality and without estimating parameters beyond those required by our approach.

Mechanism	**Examples in the literature**	**Additional functional-form or parameter restrictions**
Quasi-rent sharing	Manasse and Turrini (2001) Card et al. (2014) Card et al. (2018) Nigai (2023)	No
Fair wages	Egger and Kreickemeier (2009) Amiti and Davis (2012) Egger and Kreickemeier (2012) Egger et al. (2013)	Yes
Search and matching	Helpman et al. (2010) Helpman et al. (2017)	Yes
Efficiency wages	Davis and Harrigan (2011)	Yes
Skill assignment	Monte (2011)	Yes
Monopsony power	Card et al. (2018) Egger et al. (2021) Jha and Rodriguez-Lopez (2021)	Yes

Notes: Column 1 reports the mechanisms that can lead to linear profit sharing. Column 2 lists examples in the literature for each category. Column 3 indicates if imposing additional assumptions is required.

Table 1: Possible Microfoundations of Linear Profit Sharing

In the first column of Table 1, we list five additional mechanisms that could be used to microfound equation (15): fair wages, search and matching frictions, efficiency wages, skill assignment, and monopsonistic labor markets. While they do not represent all possible microfoundations of linear profit sharing, they cover many existing frameworks that have been used in the literature to model inequality. In the second column, we offer a non-exhaustive list of papers that use the respective mechanisms. The third column indicates whether we need to make additional assumptions and/or adopt certain functional forms to arrive at linear profit sharing in equilibrium.

In Appendix B, we provide derivation details and the required modifications to existing frameworks in Table 1 that would guarantee linear profit shifting. Here, we briefly discuss the common features of five alternative income-inequality mechanisms that are essential for our results. All five frameworks rely on two common features: (i) there is a positive assortative matching between high-income individuals and high-profit firms, and (ii) incomes are (at least partially) determined by profits.

For example, in the models based on fair wages and efficiency wages, high-profit firms pay higher wages because workers provide a maximum level of effort only if they are paid the reference wage, which is tied to profits. Models based on search and matching frictions and skill assignment lead to positive assortative matching between more productive firms and more productive individuals. In these classes of models, incomes are tied to profits via Nash bargaining. Finally, in models based on monopsonistic labor markets, firms face upward-sloping labor supply such that larger and more profitable firms must pay higher wages to attract workers. Given these common features, the conditions that we specify in Appendix B only require one additional restriction such that incomes depend on profits in a linear way.

In Appendix B, we also discuss how our formula can relate to models with firm-specific production wages and non-linear profit sharing. Given the results in Table 1, we conclude that while our baseline approach is intentionally parsimonious, it is able to provide quantitative results in a relatively isomorphic way.

4 Data Applications

In this section, we apply our approach using data from 43 countries and the Rest of the World over the period between 2000 and 2014. To calculate the effects of globalization on inequality, we need data on trade flows X_{ij}, exporter shares $(1-\rho^*_{ij})$, and domestic operating shares $(1-\rho^*_{ii})$.

First, we use total trade flows from the World Input-Output Database (Timmer et al., 2015). To obtain the share of exporters, we compile data on the number of exporter firms at the bilateral level and the total number of domestic firms. For the number of exporters at the bilateral level, we combine information from the Trade by Enterprise Characteristics database of the Organisation for Economic Cooperation and Development (OECD, 2021) and the Exporter Dynamics Database of the World Bank (World Bank, 2015). For the number of total firms, we use aggregate firm data from UNIDO Industrial Statistics Database (United Nations, 2023) and Structural and Demographic Business Statistics (SDBS) (OECD, 2023) (further details are provided in Appendix C). We observe exporter shares in 2014; this gives us the maximum number of bilateral observations on ρ^*_{ij}. Finally, following Adao et al. (2020) we measure domestic operating shares $(1 - \rho^*_{ii})$ by calculating the survival rate of manufacturing firms from the SDBS.[6] Further details on the data are provided in Appendix C.

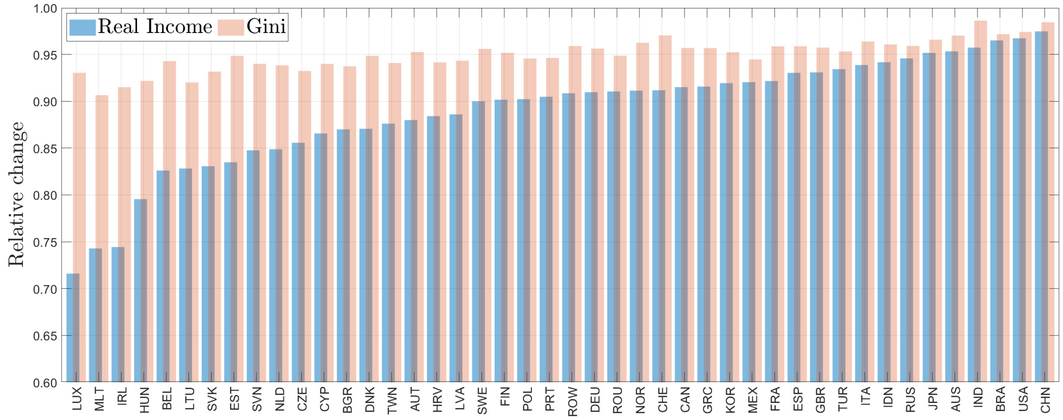
Lastly, we need the value of the parameters $\{\sigma, \theta, \alpha\}$. Following Eaton et al. (2011) (who use firm-level data from France), we set $\sigma = 3$ and $\theta = 2.5$. To calculate relative changes in the Gini coefficients, we do not need the value of α. For other measures of inequality, we use $\alpha = 1$ in the benchmark calibration. However, we show in Appendix D that the results are robust to using alternative values of $\{\sigma, \theta, \alpha\}$.

We introduce two types of counterfactual analyses using the sufficient statistics approach. First, we follow Arkolakis et al. (2012) and calculate counterfactual changes in real wage and inequality that would occur if countries reverted to autarky.[7]. We plot predictions of the sufficient statistics approach in Figure 2. The results suggest that all countries would lose in terms of aggregate welfare if they went back to autarky. Inequality, however, would also decrease.

The average country in our sample would experience an 11.0% and 5.0% decrease in real income and the Gini coefficient, respectively. Hence, the sufficient statistics approach confirms that on average globalization has been partly responsible to rising inequality. It is also worth noting that there is certain heterogeneity in the results across countries. While relatively smaller and open economies, e.g., Ireland would see significant reductions in both welfare and inequality, the effects would be less acute in larger countries with relatively high intra-trade shares, e.g., China.

[6] In the steady state of a Melitz model, the domestic operating share can be measured as the domestic survival rate as in Adao et al. (2020).

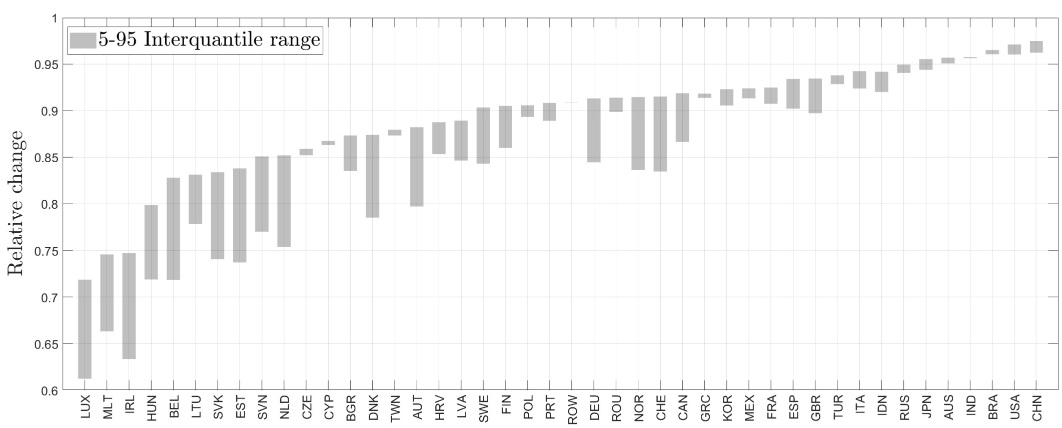
[7] To calculate counterfactual outcomes, we need counterfactual values ρ'_{ii}. We show in Appendix A that the structure of the model leads to the following identity $(1 - \rho^{*\prime}_{ii}) = (1 - \rho^*_{ii})\lambda'_{ii}/\lambda_{ii}$



Notes: The figure plots relative changes in real income and Gini coefficients if each country reverted to autarky.

Figure 2: Counterfactual Changes in Welfare and Gini Relative to Autarky

In Figure 3, we use the formula in equation (9) to investigate the distributional effects of going back to autarky by plotting the 5-95 interquartile range by country. The results suggest that for every country in our sample, the income of the 5th percentile decreases relatively less than the income of the 95th percentile such that going back to autarky would decrease inequality, which mirrors the positive effects of higher trade on inequality. On average, the 5th and 95th percentiles would lose 10.8% and 15.0% of real income, respectively.



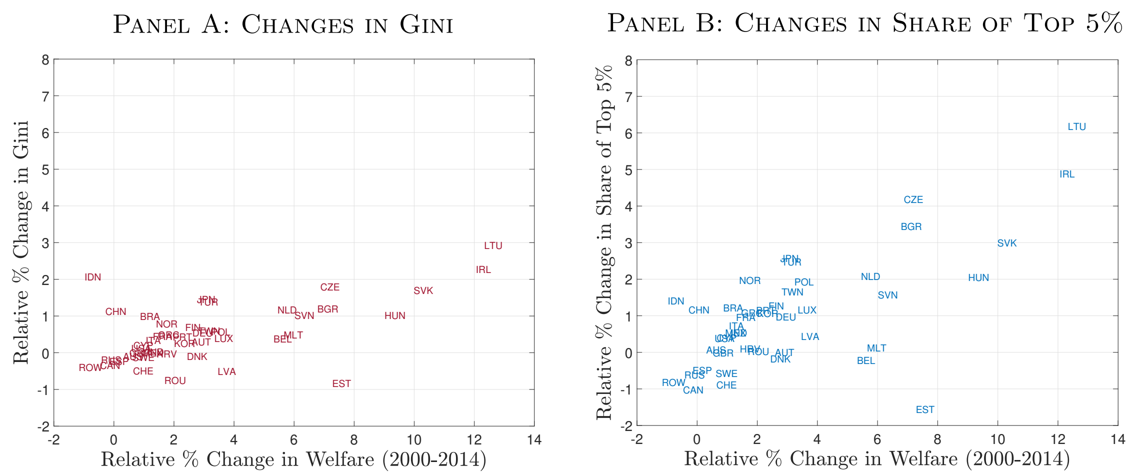
Notes: The figure plots changes in real income for the 5th and 95th quantiles as the 5-95 interquartile range relative to autarky.

Figure 3: Counterfactual Changes in 5-95 Income Interquartile Range Relative to Autarky

In our second counterfactual experiment, we use the sufficient statistics approach to calculate how inequality measures changed between 2000 and 2014. One potential concern about quantifying changes in inequality across different time periods is that changes in sufficient statistics may be due to changes in various model parameters. To address this concern, we adopt the difference-in-difference approach inspired by Adao et al. (2017). We first calculate counterfactual outcomes relative to autarky in each year and then compare the differences in the normalized outcomes over time. For example, we calculate relative changes in the Gini coefficient in percent between 2000 and 2014 as follows:

$$\Delta \mathcal{G}_i = 100 \cdot \left(\frac{\mathcal{G}_{i,2014}}{\mathcal{G}_i^{autarky}} - \frac{\mathcal{G}_{i,2000}}{\mathcal{G}_i^{autarky}} \right).$$

Figure 4, Panel A plots implied relative trade-induced changes in the Gini coefficient against welfare between 2000 and 2014 by country. Panel B presents relative changes in income shares of the top 5 percent of the population.



Notes: This figure plots the effects of trade on relative changes in inequality measures and average welfare over the 2000-2014 period for 43 countries. Panels A and B show the changes in average welfare versus the change in the Gini coefficient and the income shares of the top 5 percent of the population, respectively.

Figure 4: The Effects of Trade on Inequality and Average Welfare Over 2000-2014

Although the results are subject to a certain degree of heterogeneity across countries, they suggest that in most countries there were trade-induced increases in both measures of inequality. Countries that became more globalized during the sample period also tend to have experienced more increases in inequality. On average, the Gini coefficient grew by 0.53%, whereas the share of top 5 percent grew by 1.06%.

The results in this section suggest that the proposed sufficient approach complements the ACR formula as it allows us to calculate changes in inequality relative to autarky as well as between different time periods, while preserving the relative parsimony of the quantitative approach.

5 Extensions

In this section, we offer several extensions of the baseline model in Section 1. We discuss how the sufficient statistics can be modified to include (1) a non-tradable sector, (2) input-output linkages, (3) unemployment, and (4) heterogeneous firm-ownership shares.

5.1 Non-tradable sector

So far, we have considered a single sector model. In this subsection, we provide an extension that features an additional non-tradable (non-manufacturing) sector. As Eaton and Kortum (2002) note, this extension helps to position the manufacturing sector in the economy. As before, our goal here is to take an approach that relies on sufficient statistics to express changes in inequality.

The utility function now incorporates two sectors:

$$U_i = (Q_i^m)^{\beta_i}(Q_i^n)^{1-\beta_i},$$

where m and n superscripts denote manufacturing and non-tradable sectors; Q_i^m and Q_i^n are CES aggregates of different varieties in each sector. This means that the aggregate expenditures on output from sectors n and m are $\beta_i L_i y_i$ and $(1-\beta_i)L_i y_i$, respectively.

We assume that the number of potential entrepreneurs is fixed for each sector and that there are two sector-specific distributions of entrepreneurial ability. Entrepreneurs in one sector cannot choose to start firms in the other sector. They, however, can work and receive wages for their labor endowment in any sector. Following the same steps as in the baseline model, we can derive average profits in the manufacturing sector as:

$$\int_{\rho^s} \Pi_i^s(\rho^s)d\rho^s = \frac{\sigma^s - 1}{\sigma^s\theta^s}\frac{1}{L_i^s}\sum_j X_{ij}^s = \beta_i\frac{\sigma^s - 1}{\sigma^s\theta^s}\frac{y_i}{s_i^m},$$

where we use the trade balance condition in the manufacturing sector and s_i^m to denote the share of potential entrepreneurs in that sector. For simplicity, we assume that $\alpha = 1$ such that the average incomes in the manufacturing and non-tradable sectors can be derived as:

$$y_i^m = w_i + \beta_i \frac{\sigma^m - 1}{\sigma^m \theta^m} \frac{y_i}{s_i^m} \quad \text{and} \quad y_i^n = w_i + (1-\beta_i) \frac{\sigma^n - 1}{\sigma^n \theta^n} \frac{y_i}{(1 - s_i^m)}.$$

This means that wage can be derived as a function of the average income in the economy:

$$w_i = y_i \left(1 - \beta_i \frac{\sigma^m - 1}{\sigma^m \theta^m} - (1-\beta_i) \frac{\sigma^n - 1}{\sigma^n \theta^n}\right).$$

Next, we can derive percentile incomes in the manufacturing sector relative to the average income in the economy as:

$$\frac{y_i^m(\rho^m)}{y_i} = \left(1 - \beta_i \frac{\sigma^m - 1}{\sigma^m \theta^m} - (1-\beta_i) \frac{\sigma^n - 1}{\sigma^n \theta^n}\right) + \sum_j \mathbb{1}_{\rho > \rho_{ij}^{m*}} \frac{\beta_i}{s_i^m} \lambda_{ij} \frac{L_j y_j}{L_i y_i} \Lambda_{ij}^m(\rho^m),$$

where $\Lambda_{ij}^m(\rho^m) = \frac{\theta^m - (\sigma^m - 1)}{\sigma^m \theta^m} \frac{1}{(1 - \rho_{ij}^{m*})} \left((1-\rho^m)^{\frac{1-\sigma^m}{\theta^m}} (1 - \rho_{ij}^{m*})^{\frac{\sigma^m - 1}{\theta^m}} - 1\right)$. In a similar way, we derive percentiles in the non-tradable sector relative to the average income as:

$$\frac{y_i^n(\rho^m)}{y_i} = \left(1 - \beta_i \frac{\sigma^m - 1}{\sigma^m \theta^m} - (1-\beta_i) \frac{\sigma^n - 1}{\sigma^n \theta^n}\right) + \frac{1 - \beta_i}{1 - s_i^m} \Lambda_i^n(\rho^n),$$

where $\Lambda_i^n(\rho^n) = \frac{\theta^n - (\sigma^n - 1)}{\sigma^n \theta^n} \frac{1}{(1 - \rho_{ii}^{n*})} \left((1-\rho^n)^{\frac{1-\sigma^n}{\theta^n}} (1 - \rho_{ii}^{n*})^{\frac{\sigma^n - 1}{\theta^n}} - 1\right)$. Note that we can specify relative percentile incomes in both sectors using observable statistics as before. Relative to the baseline model, however, we need to use the following parameters $\{\beta_i, s_i^m, \sigma^n, \theta^n\}$.

It is straightforward to derive within-sector inequality following the same steps as in the baseline case. The Gini coefficient in the manufacturing sector is as follows:

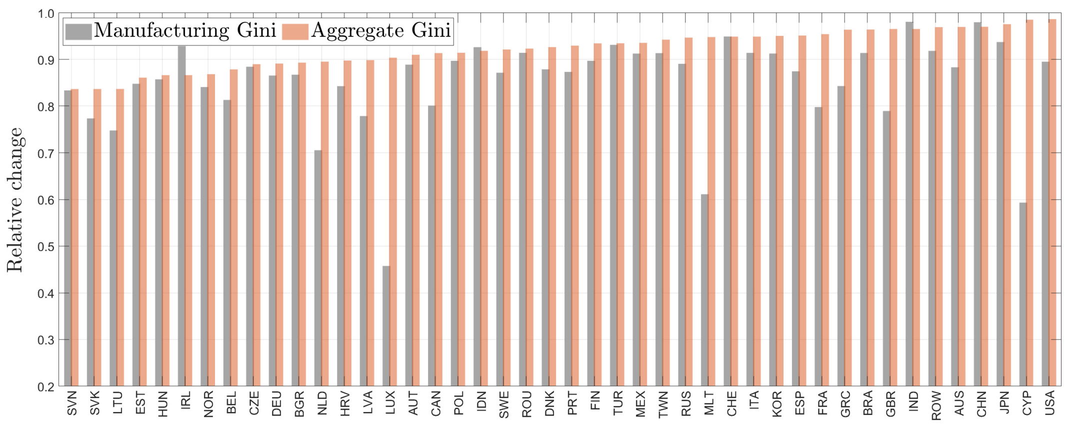
$$\mathcal{G}_i^m = \beta_i \frac{\sigma^m - 1}{\sigma^m \theta^m} + (1-\beta_i) \frac{\sigma^n - 1}{\sigma^n \theta^n} - \frac{\theta^m - (\sigma^m - 1)}{\sigma^m \theta^m} \frac{\sigma^m - 1}{2\theta^m - (\sigma^m - 1)} \frac{\beta_i}{s_i^m} \left(\sum_j \lambda_{ij} (1 - \rho_{ij}^{m*}) \frac{L_j y_j}{L_i y_i}\right),$$

and the Gini in the non-tradable sector is:

$$\mathcal{G}_i^n = \beta_i \frac{\sigma^m - 1}{\sigma^m \theta^m} + (1-\beta_i) \frac{\sigma^n - 1}{\sigma^n \theta^n} - \frac{\theta^m - (\sigma^m - 1)}{\sigma^m \theta^m} \frac{\sigma^m - 1}{2\theta^m - (\sigma^m - 1)} \frac{1 - \beta_i}{1 - s_i^m} (1 - \rho_{ii}^{n*}).$$

It is a well-known shortcoming that combining multiple Gini coefficients into a single measure in a consistent way is not possible. Hence, while we can look at within-country Gini's analytically, the overall Gini coefficient must be calculated numerically. However, the proposed sufficient approach is still useful here because we can use the expressions for percentile relative incomes in both sectors. In this case, calculating aggregate inequality measures is a straightforward computational exercise.

To quantify changes in inequality in a two-sector model that features a non-tradable sector, we calculate β_i and s_i^m from the World Input-Output Database in 2014 using consumption and employment data and assume that exit rates are identical in two sectors. We also follow Caliendo et al. (2015) and set $\sigma^n = 2.8$ and $\theta^n = 2.7$ for the non-tradable sector. We also recalculate λ_{ij} and Y_i that now reflect trade flows in the manufacturing sector.[8]



Notes: The figure plots relative changes in sectoral and aggregate Gini coefficients if each country reverted to autarky.

Figure 5: Counterfactual Changes in Sectoral and Aggregate Inequality

Figure 5 reports counterfactual changes in inequality if countries reverted back to autarky in the manufacturing sector and in the economy overall. First, quantitative results in terms of aggregate Gini's are similar to those in the baseline model. As in the baseline results, the average change in the Gini coefficients would be equal to approximately 8% in the two-sector model. On the other hand, changes in the manufacturing sector would be more acute, i.e., on average the Gini coefficients would decrease by 15%, which can be explained by relatively higher values of international trade shares, λ_{ij}^m, in the manufacturing sector.

5.2 Input-Output Linkages

In this subsection, we consider another extension of the baseline model that includes input-output linkages. Let us use s to denote sectors in the economy. Consumers have

[8] In terms of the World Input-Output Database (release 2016) classification, we classify sectors from 1 through 22 as manufacturing.

an upper-tier Cobb-Douglas utility function of the following form:

$$U_i = \prod_s \left(Q_i^s\right)^{\varsigma^s} \quad \text{, where } \sum_s \varsigma^s = 1.$$

This means that consumers spend $\varsigma^s L_i y_i$ on goods from sector s. Firms now employ labor and intermediate goods sourced via input-output linkages as in Caliendo and Parro (2014) such that the unit cost bundle is as follows:

$$c_i^s = C^s w_i^{\chi^s} \left(\prod_{\dot{s}} \left[P_i^{\dot{s}}\right]^{\varrho^{\dot{s}s}} \right)^{1-\chi^s} \text{, where } \sum_{\dot{s}} \varrho^{\dot{s}s} = 1$$

and C^s is a constant. Parameters $\chi^s \in [0,1]$ and $\varrho^{\dot{s}s}$ govern the share of value added and the intensity of sector-$\dot{s}$ output used in the production of sector s, respectively.

Now total demand for output of sector s is comprised of final and intermediate demands. Let Y_i^s denote total expenditure on goods from sector s in country i. It can be derived as follows:

$$Y_i^s = \varsigma^s L_i y_i + \sum_{\dot{s}} \varrho^{s\dot{s}} \frac{\sigma^{\dot{s}} - 1}{\sigma^{\dot{s}}} \sum_j \lambda_{ij}^{\dot{s}} Y_j^{\dot{s}}.$$

As in the previous section, we assume that individuals can supply their labor endowment in any sector. However, they can start firms only in one sector such that in each sector s there is a fixed measure of potential entrants L_i^s. While this assumption about mobility may seem restrictive, it is required to ensure there is an interior solution, i.e., production does not concentrate in a single sector (for details, see Kucheryavyy et al., 2023).

For simplicity, we assume that $\alpha = 1$. In this case, the expression for nominal income in sector s is as follows:

$$y_i^s\left(\rho^s\right) = w_i + \Pi_i^s(\rho^s),$$

where the expression for profits can be derived as:

$$\Pi_i^s(\rho^s) = \sum_j \mathbb{1}_{\rho > \rho_{ij}^{s*}} \left\{ \frac{X_{ij}^s}{L_i^s} \frac{\theta^s - (\sigma^s - 1)}{\sigma^s \theta^s} \frac{1}{(1-\rho_{ij}^*)} \left[\left(\frac{1-\rho^s}{1-\rho_{ij}^{s*}} \right)^{\frac{1-\sigma^s}{\theta^s}} - 1 \right] \right\}.$$

Following the same steps as in the baseline model, we can derive the expression for nominal income as:

$$y_i^s(\rho^s) = y_i^s - \frac{\sigma^s-1}{\sigma^s\theta^s}\frac{\sum_j X_{ij}^s}{L_i^s} + \sum_j \mathbb{1}_{\rho>\rho_{ij}^{s*}}\left\{\frac{X_{ij}^s}{L_i^s}\frac{\theta^s-(\sigma^s-1)}{\sigma^s\theta^s}\frac{1}{(1-\rho_{ij}^*)}\left[\left(\frac{1-\rho^s}{1-\rho_{ij}^{s*}}\right)^{\frac{1-\sigma^s}{\theta^s}}-1\right]\right\}.$$

One can then derive sectoral Gini coefficients following the same steps as before:

$$\mathcal{G}_i^s = \frac{\sigma^s-1}{\sigma^s\theta^s}\frac{\sum_j X_{ij}^s}{L_i^s y_i^s} - \frac{\theta^s-(\sigma^s-1)}{\sigma^s\theta^s}\frac{\sigma^s-1}{2\theta-(\sigma^s-1)}\left(\sum_j \frac{X_{ij}^s}{L_i^s y_i^s}(1-\rho_{ij}^{s*})\right).$$

Note that the Gini coefficient can still be calculated using sufficient statistics and a set of sector-specific parameters. Hence, our approach can be implemented to measure the effects of trade on within-sector inequality. Since we have the expression for nominal incomes in each sector, $y_i^s(\rho^s)$, it is also possible to derive aggregate measures of inequality. As demonstrated in Section 5.1, one would have to rely on a combined two-step approach: (i) calculate nominal incomes $y_i^s(\rho^s)$ using the sufficient statistics approach and then (ii) use sectoral income distributions to calculate aggregate inequality measures numerically.

5.3 Unemployment

Our approach can also be extended to account for unemployment. Let $\dot{\rho}_i$ be the share of unemployed individuals. Let y_i^u denote unemployment benefits provided by the government. Also, let us use y_i^e to denote average income of employed individuals. Individuals now face a joint decision of becoming entrepreneurs and supplying their production labor such that unemployment will be positive if $y_i^u > w_i$. The average income in the economy can be specified as follows:

$$y_i = \dot{\rho}_i y_i^u + (1-\dot{\rho}_i)y_i^e.$$

To finance unemployment benefits, the government collects t_i income share of working individuals as taxes. Following the same steps as in the baseline model, we can derive average profits as follows:

$$\int_\rho \Pi_i(\rho)d\rho = \frac{\sigma-1}{\sigma\theta}\sum_j \frac{X_{ij}}{(1-\dot{\rho}_i)L_i} = \frac{\sigma-1}{\sigma\theta}\frac{1}{(1-\dot{\rho}_i)}y_i. \tag{16}$$

Note that when unemployment is positive, $\dot{\rho}$ is the relevant cut-off for domestic entry.

Next, we can write wages as a function of $\dot{\rho}_i$ and y_i as:

$$w_i = \frac{1}{1-\dot{\rho}_i}\left(1-\frac{\sigma-1}{\sigma\theta}\right)y_i. \tag{17}$$

Using equations (16) and (17), we can write average incomes of unemployed and employed individuals as follows:

$$y_i^e = \frac{1-t_i}{1-\dot{\rho}_i}y_i \text{ and } y_i^u = \frac{t_i}{\dot{\rho}_i}y_i.$$

Here we assume that we can observe two sufficient statistics $\dot{\rho}_i$ and t_i. The former is simply the unemployment rate in the economy, whereas the latter can be inferred from calculating average unemployment benefit relative to the average income. Hence, we can write relative income as a function of sufficient statistics as:

$$r_i(\rho) = \begin{cases} \dfrac{t_i}{\dot{\rho}_i} & \text{for } \rho \leq \dot{\rho}_i \\ \dfrac{1-t_i}{1-\dot{\rho}_i}\left(1-\dfrac{\sigma-1}{\sigma\theta}\right) + \dfrac{1-t_i}{1-\dot{\rho}_i}\dfrac{\theta-(\sigma-1)}{\sigma\theta}\sum_j \mathbb{1}_{\rho>\rho_{ij}^*}\Lambda_{ij}^r & \text{for } \rho > \dot{\rho}_i, \end{cases}$$

where $\Lambda_{ij}^r = \lambda_{ij}\frac{L_j y_j}{L_i y_i}(1-\rho_{ij}^*)^{-1}\left[(1-\rho_{ij}^*)^{\frac{\sigma-1}{\theta}}(1-\rho)^{\frac{1-\sigma}{\theta}}-1\right]$. The Lorenz curve can then be specified as:

$$\mathcal{L}_i(\rho) = \begin{cases} \left(\dfrac{t_i}{\dot{\rho}_i}\right)\rho & \text{for } \rho \leq \dot{\rho}_i \\ t_i + (\rho-\dot{\rho}_i)\dfrac{1-t_i}{1-\dot{\rho}_i}\left(1-\dfrac{\sigma-1}{\sigma\theta}\right) + \dfrac{1-t_i}{1-\dot{\rho}_i}\dfrac{\theta-(\sigma-1)}{\sigma\theta}\sum_j \mathbb{1}_{\rho>\rho_{ij}^*}\Lambda_{ij}(\rho) & \text{for } \rho > \dot{\rho}_i, \end{cases}$$

where $\Lambda_{ij}(\rho)$ is defined as in the baseline model. Lastly, we use the expression for the Lorenz curve to derive the sufficient statistics for the Gini coefficient:

$$\mathcal{G}_i = 1 - t_i\,(2-\dot{\rho}_i) - (1-t_i)(1-\dot{\rho}_i)\left(1-\frac{\sigma-1}{\sigma\theta}\right) - \frac{1-t_i}{1-\dot{\rho}_i}\Upsilon(\theta,\sigma)\sum_j \lambda_{ij}\frac{L_j y_j}{L_i y_i}(1-\rho_{ij}^*),$$

where $\Upsilon(\theta,\sigma) = \frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{2\theta-(\sigma-1)}$. Note that relative to the baseline case, we need to observe two additional variables, i.e., unemployment rate and unemployment benefits as a share of the average income. As $t_i \to 0$ and $\dot{\rho}_i \to 0$, the formula converges back to the baseline case.

5.4 Heterogeneous ownership shares

In the baseline model, we assume that all individuals receive equal d_i as dividends. However, our approach is able to provide results for an arbitrary firm ownership structure. In this subsection, we extend our main result to account for a possible heterogeneity in dividends across individuals.

Let $s_i(\rho)$ denote the share of total dividends that accrue to individuals ρ such that $\int_0^1 s_i(\rho) = 1/L_i$. In this case, her dividends can be written down as:

$$d_i(\rho) = s_i(\rho) L_i d_i,$$

where d_i still denotes average dividends as in the baseline model. In this case, the expression for total nominal income of ρ can be written as:

$$y_i(\rho) = y_i \left(1 - \frac{(\sigma-1)}{\sigma\theta}\right) + \alpha \Pi_i(\rho) + (1-\alpha) s_i(\rho) L_i d_i.$$

This means that the relative income can be written as:

$$r_i(\rho) = \left(1 - \frac{\sigma-1}{\sigma\theta}\right) + \alpha \frac{\theta - (\sigma-1)}{\sigma\theta} \sum_j \mathbb{1}_{\rho > \rho_{ij}^*} \Lambda_{ij}^r(\rho) + (1-\alpha) \frac{\sigma-1}{\sigma\theta} s_i(\rho) L_i.$$

Hence, in principle if one had information on $s_i(\rho)$, it would be possible to apply our approach. We consider one example, where we assume that $s_i(\rho)$ is increasing in ρ as follows:

$$s_i(\rho) = \frac{1}{L_i} \ell \rho^{\varphi},$$

where $\ell/(\varphi+1) = 1$ such that $\int_0^1 s_i(\rho) = 1/L_i$. In this case, the Lorenz curve is going to be given by the following expression:

$$\mathcal{L}_i(\rho) = \left(1 - \frac{\sigma-1}{\sigma\theta}\right) \rho + \alpha \frac{\theta - (\sigma-1)}{\sigma\theta} \sum_j \mathbb{1}_{\rho > \rho_{ij}^*} \Lambda_{ij}(\rho) + (1-\alpha) \frac{\sigma-1}{\sigma\theta} \rho^{\varphi+1},$$

and the expression for the Gini coefficient is as follows:

$$\mathcal{G}_i = \frac{\sigma-1}{\sigma\theta} \left(1 - \frac{2(1-\alpha)}{\varphi+2}\right) - \alpha \frac{\theta-(\sigma-1)}{\sigma\theta} \frac{\sigma-1}{2\theta-(\sigma-1)} \left(\sum_j \lambda_{ij}(1-\rho_{ij}^*) \frac{L_j y_j}{L_i y_i}\right). \quad (18)$$

Note that the Gini coefficient will be higher than in the baseline model if the following condition holds:

$$1 - \frac{2(1-\alpha)}{\varphi + 2} > \alpha \quad \Leftrightarrow \quad (1-\alpha)\varphi > 0.$$

This means that as long as there are dividends to distribute ($\alpha < 1$) and higher ρ's own larger shares of firms ($\varphi > 0$), inequality will be higher relative to the baseline case.

6 Conclusions

In this paper, we have developed a parsimonious way of quantifying changes in key inequality measures. Our method both complements and remains consistent with the standard ACR formula. A key strength of our approach is that one can derive changes in inequality without explicitly modeling inequality mechanisms. Instead, our results rely on observing two sufficient statistics: aggregate bilateral trade flows and the share of exporters.

The sufficient statistics perspective in this paper relies on the analytical expression for income levels across the entire distribution relative to the average. This allows us to derive analytical expressions for the Lorenz curve, the Gini coefficient, and the total income shares held by different population groups. We apply our method to the data covering 43 countries over the period from 2000 to 2014. We find that reverting to autarky relative to 2014 would decrease the Gini coefficient by 5% on average. We also find that, for an average country between 2000 and 2014, the Gini coefficient increased by 0.53%, and the top 5 percent share of income increased by 1.06% due to trade. We are also able to describe distributional welfare gains from trade in terms of interquantile ranges.

We also provide several extensions that allow us to incorporate the sufficient statistics approach in broader environments including a non-tradable sector, input-output linkages, unemployment, and heterogeneous ownership shares. Overall, we consider the proposed approach to offer a transparent and useful way of calculating the effects of trade on inequality with minimal data requirements.

References

Adao, Rodrigo, Arnaud Costinot, and Dave Donaldson, "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade," *American Economic Review*, March 2017, *107* (3), 633–89.

Adao, Rodrigo, Costas Arkolakis, and Sharat Ganapati, "Aggregate Implications of Firm Heterogeneity: A Nonparametric Analysis of Monopolistic Competition Trade Models," NBER Working Papers 28081, National Bureau of Economic Research, Inc November 2020.

_ , Paul Carrillo, Arnaud Costinot, Dave Donaldson, and Dina Pomeranz, "Imports, Exports, and Earnings Inequality: Measures of Exposure and Estimates of Incidence," *The Quarterly Journal of Economics*, 03 2022, *137* (3), 1553–1614.

Amiti, Mary and Donald R. Davis, "Trade, Firms, and Wages: Theory and Evidence," *Review of Economic Studies*, 2012, *79* (1), 1–36.

Anderson, James E., Mario Larch, and Yoto V. Yotov, "GEPPML: General equilibrium analysis with PPML," *The World Economy*, 2018, *41* (10), 2750–2782.

Antras, Pol, Alonso de Gortari, and Oleg Itskhoki, "Globalization, inequality and welfare," *Journal of International Economics*, 2017, *108* (C), 387–412.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodriguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, February 2012, *102* (1), 94–130.

_ , _ , Dave Donaldson, and Andres Rodriguez-Clare, "The Elusive Pro-Competitive Effects of Trade," *Review of Economic Studies*, 2019, *86* (1), 46–80.

Artuc, Erhan, Guido Porto, and Bob Rijkers, "Trading off the income gains and the inequality costs of trade policy," *Journal of International Economics*, 2019, *120*, 1–45.

Autor, David H., David Dorn, Gordon H. Hanson, and Jae Song, "Trade Adjustment: Worker-Level Evidence," *The Quarterly Journal of Economics*, 09 2014, *129* (4), 1799–1860.

Burstein, Ariel and Jonathan Vogel, "International Trade, Technology, and the Skill Premium," *Journal of Political Economy*, 2017, *125* (5), 1356–1412.

Caliendo, Lorenzo and Fernando Parro, "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 11 2014, *82* (1), 1–44.

_ , Robert C Feenstra, John Romalis, and Alan M Taylor, "Tariff Reductions, Entry, and Welfare: Theory and Evidence for the Last Two Decades," Working Paper 21768, National Bureau of Economic Research December 2015.

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline, "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 2018, *36* (S1), S13–S70.

_ , Francesco Devicienti, and Agata Maida, "Rent-sharing, Holdup, and Wages: Evidence from Matched Panel Data," *The Review of Economic Studies*, 2014, *81* (1 (286)), 84–111.

Chaney, Thomas, "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, September 2008, *98* (4), 1707–1721.

Costinot, Arnaud and Andres Rodriguez-Clare, "Chapter 4 - Trade Theory with Numbers: Quantifying the Consequences of Globalization," in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4 of *Handbook of International Economics*, Elsevier, 2014, pp. 197–261.

Davis, Donald R. and James Harrigan, "Good jobs, bad jobs, and trade liberalization," *Journal of International Economics*, May 2011, *84* (1), 26–36.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum, "Unbalanced Trade," *American Economic Review*, May 2007, *97* (2), 351–355.

Dhyne, Emmanuel, Ayumu Ken Kikkawa, Toshiaki Komatsu, Magne Mogstad, and Felix Tintelnot, "Foreign Demand Shocks to Production Networks: Firm Responses and Worker Impacts," Working Paper 30447, National Bureau of Economic Research September 2022.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, September 2002, *70* (5), 1741–1779.

_ , _ , and Francis Kramarz, "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 2011, *79* (5), 1453–1498.

Egger, Hartmut and Udo Kreickemeier, "Firm Heterogeneity And The Labor Market Effects Of Trade Liberalization," *International Economic Review*, February 2009, *50* (1), 187–216.

_ and _ , "Fairness, trade, and inequality," *Journal of International Economics*, 2012, *86* (2), 184–196.

_ , Peter Egger, and Udo Kreickemeier, "Trade, wages, and profits," *European Economic Review*, 2013, *64*, 332–350.

_ , Udo Kreickemeier, Christoph Moser, and Jens Wrona, "Exporting and Offshoring with Monopsonistic Competition," *The Economic Journal*, 10 2021, *132* (644), 1449–1488.

Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer, "The Next Generation of the Penn World Table," *American Economic Review*, October 2015, *105* (10), 3150–82.

Felbermayr, Gabriel, Julien Prat, and Hans-Jörg Schmerer, "Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity," *Journal of Economic Theory*, January 2011, *146* (1), 39–73.

Galle, Simon, Andres Rodriguez-Clare, and Moises Yi, "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade," *The Review of Economic Studies*, 04 2022, *90* (1), 331–375.

Harrigan, James and Ariell Reshef, "Skill-biased heterogeneous firms, trade liberalization and the skill premium," *Canadian Journal of Economics*, August 2015, *48* (3), 1024–1066.

Helpman, Elhanan and Oleg Itskhoki, "Labour Market Rigidities, Trade and Unemployment," *Review of Economic Studies*, 2010, *77* (3), 1100–1137.

_ , _ , and Stephen Redding, "Inequality and Unemployment in a Global Economy," *Econometrica*, July 2010, *78* (4), 1239–1283.

_ , _ , Marc-Andreas Muendler, and Stephen J. Redding, "Trade and Inequality: From Theory to Estimation," *Review of Economic Studies*, 2017, *84* (1), 357–405.

Hummels, David, Rasmus Jorgensen, Jakob Munch, and Chong Xiang, "The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data," *American Economic Review*, June 2014, *104* (6), 1597–1629.

Jha, Priyaranjan and Antonio Rodriguez-Lopez, "Monopsonistic labor markets and international trade," *European Economic Review*, 2021, *140*, 103939.

Jones, Ronald W., "The Structure of Simple General Equilibrium Models," *Journal of Political Economy*, 1965, *73* (6), 557–572.

Kim, Ryan and Jonathan Vogel, "Trade Shocks and Labor Market Adjustment," *American Economic Review: Insights*, March 2021, *3* (1), 115–30.

Kucheryavyy, Konstantin, Gary Lyn, and Andres Rodriguez-Clare, "Grounded by Gravity: A Well-Behaved Trade Model with Industry-Level Economies of Scale," *American Economic Journal: Macroeconomics*, April 2023, *15* (2), 372–412.

Lashkaripour, Ahmad, "The cost of a global tariff war: A sufficient statistics approach," *Journal of International Economics*, 2021, *131*, 103419.

Manasse, Paolo and Alessandro Turrini, "Trade, wages, and 'superstars'," *Journal of International Economics*, June 2001, *54* (1), 97–117.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, November 2003, *71* (6), 1695–1725.

Monte, Ferdinando, "Skill bias, trade, and wage dispersion," *Journal of International Economics*, 2011, *83* (2), 202–218.

Nigai, Sergey, "Selection Effects, Inequality, and Aggregate Gains from Trade," forthcoming, *Journal of International Economics*, 2023.

OECD, "Trade by enterprise characteristics: Trade by size classes, ISIC Rev. 4 (Edition 2020)," 2021.

_ , "Structural business statistics ISIC Rev. 4," 2023.

Parro, Fernando, "Capital-Skill Complementarity and the Skill Premium in a Quantitative Model of Trade," *American Economic Journal: Macroeconomics*, 2013, *5* (2), 72–117.

Rodrik, Dani, "A Primer on Trade and Inequality," NBER Working Papers 29507, National Bureau of Economic Research, Inc November 2021.

Sampson, Thomas, "Selection into Trade and Wage Inequality," *American Economic Journal: Microeconomics*, August 2014, *6* (3), 157–202.

Silva, J. M. C. Santos and Silvana Tenreyro, "The Log of Gravity," *The Review of Economics and Statistics*, 11 2006, *88* (4), 641–658.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries, "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production," *Review of International Economics*, 2015, *23* (3), 575–605.

United Nations, "INDSTAT2 2020 ISIC Rev.3," 2023.

World Bank, "Exporter Dynamics Database," 2015.

Yeaple, Stephen Ross, "A simple model of firm heterogeneity, international trade, and wages," *Journal of International Economics*, January 2005, *65* (1), 1–20.

Appendix A: Derivation Details

In this Appendix, we provide derivation details behind equations in the main text. Start with deriving the expression for trade flows:

$$X_{ij} = L_i b_i^{\sigma-1} V_{ij} \int_{\rho_{ij}^*}^{1} (1-\rho)^{\frac{1-\sigma}{\theta}} d\rho = L_i b_i^{\sigma-1} V_{ij} \frac{\theta}{\theta-(\sigma-1)} (1-\rho_{ij}^*)^{\frac{1-\sigma}{\theta}+1}. \tag{A1}$$

where we use an auxiliary variable $V_{ij} = \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} P_j^{\sigma-1} L_j w_j$. From the zero-profit condition, we know that the following holds:

$$(1-\rho_{ij}^*)^{\frac{1-\sigma}{\theta}} = b_i^{1-\sigma} \sigma w_j f_{ij} V_{ij}^{-1}. \tag{A2}$$

Using this identity in the expression for trade flows allows to further simplify them as:

$$X_{ij} = L_i b_i^{1-\sigma} V_{ij} \frac{\theta}{\theta-(\sigma-1)} b_i^{1-\sigma} \sigma w_j f_{ij} V_{ij}^{-1} (1-\rho_{ij}^*) = \frac{\sigma\theta}{\theta-(\sigma-1)} (1-\rho_{ij}^*) L_i w_j f_{ij}. \tag{A3}$$

Lastly, we use the expression in equation (A1)-(A3) to express trade shares as:

$$\lambda_{ij} = \frac{X_{ij}}{\sum_k X_{kj}} = \frac{L_i b_i^{\theta} (w_i \tau_{ij})^{-\theta} f_{ij}^{1-\frac{\theta}{\sigma-1}}}{\sum_k L_k b_k^{\theta} (w_k \tau_{kj})^{-\theta} f_{kj}^{1-\frac{\theta}{\sigma-1}}}.$$

In the main text, we use the result that $(1-\rho_{ii}^{*\prime}) = (1-\rho_{ii}^*)\dfrac{\lambda_{ii}'}{\lambda_{ii}}$. Here we provided derivation details. First, note that combining equations (A1), (A2), and (A3) gives the following expressions for trade flows:

$$X_{ii} = \frac{\sigma\theta}{\theta-(\sigma-1)} L_i w_i f_{ii} (1-\rho_{ii}^*).$$

We also know from equation (2) that $w_i = \frac{\sigma\theta-(\sigma-1)}{\sigma\theta} y_i$ such that the following must hold:

$$\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)} \frac{1}{f_{ii}} \lambda_{ii} = (1-\rho_{ii}^*).$$

Given that $(1-\rho_{ii}^*)$ is a linear function of λ_{ii} and parameters of the model that do not change, the following must hold:

$$(1-\rho_{ii}^{*\prime}) = (1-\rho_{ii}^*)\frac{\lambda_{ii}'}{\lambda_{ii}}.$$

Proof of Lemma 1. First, state the total profits of firm ρ:

$$\Pi_i(\rho) = \sum_j \mathbb{1}_{\rho>\rho_{ij}^*} \left\{ b_i^{\sigma-1} (1-\rho)^{\frac{1-\sigma}{\theta}} \frac{1}{\sigma} V_{ij} - b_i^{\sigma-1} (1-\rho_{ij}^*)^{\frac{1-\sigma}{\theta}} \frac{1}{\sigma} V_{ij} \right\}. \tag{A4}$$

Applying identities in (A1),(A2),(A3) and the fact that there are L_i potential entrants allows us to reformulate total profits as:

$$\Pi_i(\rho) = \sum_j \mathbb{1}_{\rho > \rho^*_{ij}} \left\{ \frac{X_{ij}}{L_i} \frac{\theta - (\sigma - 1)}{\sigma\theta} \frac{1}{(1 - \rho^*_{ij})} \left[\left(\frac{1 - \rho}{1 - \rho^*_{ij}} \right)^{\frac{1-\sigma}{\theta}} - 1 \right] \right\}. \tag{A5}$$

Next, recall the expression for total nominal income in the main text:

$$y_i(\rho) = \underbrace{w_i}_{(i)} + \underbrace{\alpha\Pi_i(\rho)}_{(ii)} + \underbrace{(1-\alpha)\int_\rho \Pi_i(\rho) d\rho}_{(iii)}.$$

Here (i) is a function of y_i as shown in equation (2) in the main text, and (ii) can be computed using the expression in (A5). To derive the expression for average profits, we derive the following integral:

$$\int_{\rho^*_{ij}}^1 \left[(1 - \rho^*_{ij})^{\frac{\sigma-1}{\theta}} (1-\rho)^{\frac{1-\sigma}{\theta}} - 1 \right] d\rho = (1 - \rho^*_{ij}) \frac{\sigma - 1}{\theta - (\sigma - 1)}. \tag{A6}$$

Then, the following must hold:

$$\int_0^1 \Pi_i(\rho) d\rho = \frac{\sigma - 1}{\sigma\theta} y_i \tag{A7}$$

Then, the following must hold:

$$d_i = (1-\alpha)\int_\rho \Pi_i(\rho) d\rho = (1-\alpha)\frac{\sigma - 1}{\sigma\theta} y_i.$$

We can derive wage as a function of the average income as follows:

$$w_i = y_i \left(1 - \frac{\sigma - 1}{\sigma\theta} \right) \tag{A8}$$

This means that total nominal income can be specified as:

$$y_i(\rho) = \left(1 - \alpha\frac{\sigma - 1}{\sigma\theta} \right) y_i + \alpha\Pi_i(\rho).$$

For convenience, let us define the following auxiliary variable:

$$\Lambda^r_{ij}(\rho) = \left\{ \lambda_{ij} \frac{L_j y_j}{L_i y_i} \frac{1}{(1 - \rho^*_{ij})} \left[\left(\frac{1 - \rho}{1 - \rho^*_{ij}} \right)^{\frac{1-\sigma}{\theta}} - 1 \right] \right\} \tag{A9}$$

Then, the nominal income of ρ relative to the average income can be specified as:

$$r_i(\rho) = \left(1 - \alpha\frac{\sigma - 1}{\sigma\theta} \right) + \alpha\frac{\theta - (\sigma - 1)}{\sigma\theta} \sum_j \mathbb{1}_{\rho > \rho^*_{ij}} \Lambda^r_{ij}(\rho) \text{ for } \rho > \rho^*_{ii}.$$

$\blacksquare$

Proof of Proposition 1. Start with the definition of the Lorenz curve:

$$\mathcal{L}_i(\rho') = \int_0^{\rho'} r_i(\rho) d\rho \text{ such that } \mathcal{L}_i(0) = 0 \text{ and } \mathcal{L}_i(1) = 1.$$

With the expression for the relative income from equation (8) in hand, we can derive the expression for the Lorenz curve as:

$$\mathcal{L}_i(\rho') = \left(1 - \alpha\frac{\sigma - 1}{\sigma\theta}\right) \int_0^{\rho'} d\rho + \alpha\frac{\theta - (\sigma - 1)}{\sigma\theta} \sum_j \mathbb{1}_{\rho > \rho^*_{ij}} \int_{\rho^*_{ij}}^{\rho'} \Lambda^r_{ij}(\rho') d\rho$$

To derive the analytical expression for the Lorenz curve, we need to take the following integrals in the above equation. The first integral is as follows:

$$\left(1 - \alpha\frac{\sigma - 1}{\sigma\theta}\right) \int_0^{\rho'} d\rho = \left(1 - \alpha\frac{\sigma - 1}{\sigma\theta}\right) \rho',$$

The second integral is given ass:

$$\int_{\rho^*_{ij}}^{\rho'} \Lambda^r_{ij}(\rho') d\rho = \sum_j \mathbb{1}_{\rho > \rho^*_{ij}} \lambda_{ij} \frac{L_j y_j}{L_i y_i} \left(\frac{\theta}{\theta - (\sigma - 1)} \left[1 - \left(\frac{1 - \rho'}{1 - \rho^*_{ij}} \right)^{\frac{1-\sigma}{\theta}+1} \right] - 1 + \left(\frac{1 - \rho'}{1 - \rho^*_{ij}} \right) \right).$$

For convenience, let us define an auxiliary variable:

$$\Lambda_{ij}(\rho') = \lambda_{ij} \frac{L_j y_j}{L_i y_i} \left(\frac{\theta}{\theta - (\sigma - 1)} \left[1 - \left(\frac{1 - \rho'}{1 - \rho^*_{ij}} \right)^{\frac{1-\sigma}{\theta}+1} \right] - 1 + \left(\frac{1 - \rho'}{1 - \rho^*_{ij}} \right) \right)$$

Then, the analytical expression for the Lorenz curve is as follows:

$$\mathcal{L}_i(\rho') = \left(1 - \alpha\frac{\sigma - 1}{\sigma\theta}\right) \rho' + \alpha\frac{\theta - (\sigma - 1)}{\sigma\theta} \sum_j \mathbb{1}_{\rho > \rho^*_{ij}} \Lambda_{ij}(\rho') \text{ for } \rho > \rho^*_{ii}$$

Next, we derive the expression for the Gini coefficient. Start with the definition:

$$\mathcal{G}_i = 1 - 2 \int_0^1 \mathcal{L}_i(\rho) d\rho,$$

where $\mathcal{L}_i(\rho)$ is given as above. To get the analytical expression for the Gini coefficient we need to find three integrals. The first integral is as follows:

$$\left(1 - \alpha\frac{\sigma - 1}{\sigma\theta}\right) \int_0^1 \rho d\rho = \left(1 - \alpha\frac{\sigma - 1}{\sigma\theta}\right) \frac{1}{2},$$

The second integral is:

$$\int_{\rho_{ij}^*}^1 \Lambda_{ij}(\rho)d\rho = \frac{\sigma-1}{4\theta-2(\sigma-1)}\left(\sum_j \lambda_{ij}(1-\rho_{ij}^*)\frac{L_j y_j}{L_i y_i}\right)$$

Hence, the following equation must hold:

$$\int_0^1 \mathcal{L}_i(\rho)d\rho = \left(1-\alpha\frac{\sigma-1}{\sigma\theta}\right)\frac{1}{2} + \alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{4\theta-2(\sigma-1)}\left(\sum_j \lambda_{ij}(1-\rho_{ij}^*)\frac{L_j y_j}{L_i y_i}\right).$$

Putting everything together yields:

$$\mathcal{G}_i = \alpha\frac{\sigma-1}{\sigma\theta} - \alpha\frac{\theta-(\sigma-1)}{\sigma\theta}\frac{\sigma-1}{2\theta-(\sigma-1)}\left(\sum_j \lambda_{ij}(1-\rho_{ij}^*)\frac{L_j y_j}{L_i y_i}\right)$$

■

Appendix B: Alternative Linear-profit-sharing mechanisms

In this Appendix, we provide derivation details and additional assumptions necessary to modify existing frameworks such that they lead to linear profit sharing. Since we discuss linear profit sharing based on quasi-rent sharing in the main text, here we focus on five alternative mechanisms: fair wages, search and matching frictions, efficiency wages, skill assignment, and monopsonistic labor markets.

B1: Fair wages

We provide another microfoundation for the profit-sharing mechanism using the concept of fair wages as in Egger and Kreickemeier (2009, 2012), Amiti and Davis (2012) and Egger et al. (2013). However, as mentioned in the main text, we make several assumptions to make this framework fall under the umbrella of linear profit shifting.

Individuals receive income from providing managerial services and selling their labor endowments. However, we now assume that they are identical in terms of their ex-ante abilities. Firms, on the other hand, are heterogeneous and are still pinned down by their place in the productivity distribution denoted by ρ. Individuals are hired as managers by firms and provide a certain level of effort according to the following equation:

$$\varepsilon_i(\rho) = \min\left\{\frac{z_i(\rho)}{\bar{z}_i(\rho)}, 1\right\},$$

where $\bar{z}_i(\rho)$ is the level of managerial income that the individual would consider *fair* and $z_i(\rho)$ is income that she actually receives from firm ρ. Managerial effort affects total productivity, which now is a product of the manager's effort and firm's fundamental productivity such that the marginal cost becomes:

$$m_{ij}(\rho) = \underbrace{\varepsilon_i(\rho)^{-1}}_{\text{Effort component}} \cdot \underbrace{(1-\rho)^{\frac{1}{\theta}}}_{\text{Firm component}} \cdot \underbrace{\left(\frac{\tau_{ij}w_i}{b_i}\right)}_{\text{Country-pair component}} .$$

As noted in Egger and Kreickemeier (2009), in equilibrium profit-maximizing firms pay exactly $\bar{z}_i(\rho)$ such that $\varepsilon(\rho) = 1$. We follow Amiti and Davis (2012) and specify the following constraints for the fair-wage function:

(i) $\bar{z}_i(\rho^*_{ii})$ is a constant

(ii) $0 < \dfrac{\partial \bar{z}_i(\rho)}{\partial \Pi_i(\rho)} < \infty$

(iii) $\bar{z}_i(\rho)$ has a finite upper bound

While there are many functions that would satisfy (i)-(iii), we propose a linear fair-wage function as our main specification and discuss the alternatives in what follows. If we assume that the fair-wage function is linear in firm's profits as follows:

$$\bar{z}_i(\rho) = \mu\Pi_i(\rho) + (1-\mu)(1-U)\int_\rho \Pi_i(\rho)d\rho,$$

where μ is the fairness parameter and $(1-U)$ is the probability that the individual can be matched with another firm if she decides to leave ρ. Here, $U \in [0, 1]$ may be interpreted as a search costs that the individual has to incur to match with another firm or the risk of unemployment. We assume that U is sufficiently high such that individuals do not have incentives to leave once they are matched. Note that in the limiting case $\mu = 1$, the specification is identical to our baseline model. When $\mu < 1$, the firm pays $\bar{z}_i(\rho)$ to the entrepreneur such that the residual profits (paid out as dividends) are:

$$d_i(\rho) = \Pi_i(\rho) - \bar{z}_i(\rho) = (1-\mu)\Pi_i(\rho) - (1-\mu)(1-U)\int_\rho \Pi_i(\rho)d\rho$$

Average dividends can then be specified as:

$$d_i = \int_\rho d_i(\rho)d\rho = (1-\mu)U\int_\rho \Pi_i(\rho)d\rho$$

Individuals still receive income w_i for their labor endowment. Hence, putting all the parts together, we can specify total income of individual employed at ρ as:

$$y_i(\rho) = w_i + \mu\Pi_i(\rho) + (1-\mu)(1-U)\int_\rho \Pi_i(\rho)d\rho + (1-\mu)U\int_\rho \Pi_i(\rho)d\rho.$$

Simplify to get:

$$y_i(\rho) = w_i + \mu\Pi_i(\rho) + (1-\mu)\int_\rho \Pi_i(\rho)d\rho. \tag{B1}$$

Equation (B1) makes it clear that the linear profit-sharing mechanism in our baseline model and the one based on the fair-wage preferences are isomorphic under the conditions that the fair-wage function is linear.

An alternative to the linear specification of the fair-wage function would be using the multiplicative function in Egger and Kreickemeier (2012); Egger et al. (2013) as follows:

$$\bar{z}_i(\rho) = [\alpha \Pi_i(\rho)]^{\mu} \left[\alpha(1-U)\int_{\rho} \Pi_i(\rho) d\rho\right]^{(1-\mu)}. \tag{B2}$$

Note that as $\mu \to 1$, equation (B2) becomes our baseline specification, which means that the fair-wage mechanism is isomorphic under the alternative set of conditions, i.e., fair-wage function is Cobb-Douglas and the fairness parameter $\mu = 1$.

B2: Search and matching

We now extend the baseline model by incorporating the search-and-matching friction as in Helpman et al. (2010) and Helpman et al. (2017). Individuals receive income from providing managerial services and selling their labor endowments. To generate incentives for manager searching, we introduce complementarity between the firm's inherent productivity and the individual's managerial ability. This means that the productivity level of a firm is a product of firm's inherent productivity and and the hired individual's managerial ability $\phi \cdot \varepsilon$.

The inherent firm productivity is still drawn from the Pareto distribution as the baseline model, and we continue to index the firm by ρ. The managerial ability ε is drawn from the Pareto distribution $G_i(\varepsilon) = 1 - \varepsilon_{i,min}^{k} \varepsilon^{-k}$ with $\varepsilon_{i,min} > 0$ and $k > 1$. Henceforth, for brevity we drop country subscript i in $\varepsilon_{i,min}$. We follow Helpman et al. (2010) and interpret the managerial ability as match-specific, and the individual's draw for a given match conveys no information about ability draws for other potential matches. Managerial ability cannot be costlessly observed. Instead, firms undertake costly investments in screening to obtain an imprecise signal of managerial ability.

To incorporate costly matching into the model, we introduce search cost and screening cost. The specifications for the search and screening costs follow Helpman et al. (2010), but we show at the end that any power function of the managerial ability cutoff as cost functions would yield the same implication. First, the search cost is $(s/k) \cdot n$ units of wages with costs s and n measure of screened candidates.[9] A firm with managerial ability cutoff ε_c screens $n = (\varepsilon_c/\varepsilon_{min})^k$ measure of candidates.

The screening cost is $(c/\kappa) \cdot \varepsilon_c^{\kappa}$ units of wages, where $c > 0$ and $\kappa > 0$. The value of c measures the level of search cost, while κ governs the convexity of the cost. Firms can identify workers with an ability below ε_c by incurring the screening cost. Screening costs are increasing in the chosen ability threshold ε_c because more complex and costlier tests are required for higher ability cutoffs. We also follow Helpman et al. (2010) and assume that firms and the matched individual only know whether her managerial ability exceeds the cutoff at the hiring and bargaining stage. The ability is only realized during the time of operation.

[9] The cost component can be endogenized with labor market tightness by following the Diamond-Mortensen-Pissarides approach. Helpman et al. (2010) use $s = \eta_0 x^{\eta_1}$ with $\eta_0 > 1$ and $\eta_1 > 0$ where x is the labor market tightness.

The timing of events proceeds as follows. First, a firm draws its productivity and chooses the level of manager cutoff, $\varepsilon_c(\rho)$, by maximizing the residual profit, which takes into account bargaining outcome and search-and-matching costs. Upon matching, the firm and its hired individual engage in strategic bargaining. As in the main model, we adopt the Nash bargaining.[10] The matched individual takes $\alpha = \gamma$ share of profits where α is the profit-sharing parameter in R4 and γ measures the bargaining power of individuals as managers. The firm takes $1-\alpha$ share of profits net of search-and-matching costs. The managerial ability and the profit are realized after the operation.

Working backward, the expected marginal cost of firm ρ from country i that serves market j at the hiring and bargaining stage is:

$$\mathbb{E}[m_{ij}(\rho)] = \underbrace{\mathbb{E}[\varepsilon(\rho)^{-1}]}_{\text{Manager component}} \cdot \underbrace{(1-\rho)^{\frac{1}{\theta}}}_{\text{Firm component}} \cdot \underbrace{\left(\frac{\tau_{ij}w_i}{b_i}\right)}_{\text{Country-pair component}}, \tag{B3}$$

where $\mathbb{E}$ is the expectation operator. To make the notation consistent with the baseline model, let us use $\tilde{\Pi}_i(\rho)$ and $\Pi_i(\rho)$ to denote total profits *net* and *gross* of search-and-matching costs, respectively. The firm's maximization problem with respect to the managerial ability cutoff is as follows:

$$\max_{\varepsilon_c(\rho)} \tilde{\Pi}_i(\rho) = \max_{\varepsilon_c(\rho)} \left\{ (1-\alpha)\mathbb{E}\left[\Pi_i(\rho)\right] - \underbrace{w_i \frac{s}{k}\left(\frac{\varepsilon_c(\rho)}{\varepsilon_{min}}\right)^k}_{\text{search cost}} - \underbrace{w_i \frac{c}{\kappa}\varepsilon_c(\rho)^\kappa}_{\text{screening cost}} \right\}.$$

By taking the first-order condition, we obtain:[11]

$$(1-\alpha)(\sigma-1)\Pi_i(\rho) = w_i s\left(\frac{\varepsilon_c(\rho)}{\varepsilon_{min}}\right)^k + w_i c\varepsilon_c(\rho)^\kappa. \tag{B4}$$

We can derive the analytical expression for $\varepsilon_c(\rho)$ when the values of k and κ are the same. By applying the (B4), we can specify total nominal income of individual ρ as:

$$\begin{aligned} y_i(\rho) &= w_i + \alpha\Pi_i(\rho) + \int_\rho \left((1-\alpha)\Pi_i(\rho) - w_i\frac{s}{k}\left(\frac{\varepsilon_c(\rho)}{\varepsilon_{min}}\right)^k - w_i\frac{c}{\kappa}\varepsilon_c(\rho)^\kappa\right)d\rho \\ &= w_i + \alpha\Pi_i(\rho) + \left((1-\alpha)\left(\frac{\kappa-(\sigma-1)}{\kappa}\right)\right)\int_\rho \Pi_i(\rho)d\rho, \end{aligned}$$

where we impose $k = \kappa$, which is a parametric restriction that we need to introduce. Note that now the portion of dividends is "wasted" due to search-and-matching friction. Following the same step as in the baseline model, we can derive the expression for wages as a function of average income:

$$w_i = \left(1 - \frac{(\sigma-1)}{\sigma\theta}\left(\alpha + (1-\alpha)\frac{\kappa-(\sigma-1)}{\kappa}\right)\right) y_i. \tag{B5}$$

[10]Helpman et al. (2010) use the Stole-Zwiebel bargaining, which is a generalization of Nash bargaining to multiple workers' cases. Each worker in their model receives a fraction of the average revenue per worker. A Firm anticipates the bargaining outcome and maximizes the share of revenue net of any fixed costs and search-and-matching costs as the firm's profit.

[11]For the first-order condition to be well-behaved, we assume that $\varepsilon_c(\rho)$ affects firms profits conditional on entry in export markets.

In this case, the sum of wages and dividends is the same as in the baseline model:

$$w_i + d_i = y_i \left(1 - \frac{\alpha(\sigma - 1)}{\sigma\theta}\right).$$

The intuition behind this result is that in relative terms when a portion of the residual profit is wasted, the dividends share of total income decreases, while the wage share of total income increases. The two cancel out so that the total relative income is maintained. Hence, we conclude that we can microfound linear profit sharing using search-and-matching frictions with the following restrictions: (i) firms and managers engage in Nash bargaining over profits; and (ii) the parameter governing the convexity of the search cost function equals the dispersion parameter of managerial ability ($k = \kappa$). The latter implies that if the ability is less dispersed, it becomes harder to distinguish high- and low-ability manager ex-ante.

B3: Efficiency wages

We provide another microfoundation for efficiency wages using the Shapiro-Stiglitz efficiency wages framework as in Davis and Harrigan (2011). We continue to assume that individuals receive income from providing managerial services and selling their labor endowments. However, in this framework, workers now have incentives to shirk due to disutilities of working.

Individuals provide a certain level of effort according to the following equation:

$$\varepsilon_i(\rho) = \min\left\{\frac{z_i(\rho)}{\bar{z}_i(\rho)}, 1\right\},$$

where $\bar{z}_i(\rho)$ is the level of managerial income that the individual would consider *high enough* to eliminate shirking and $z_i(\rho)$ is income that she actually receives from firm ρ. The marginal cost consists of the effort component and the firm's fundamental productivity:

$$m_{ij}(\rho) = \underbrace{\varepsilon_i(\rho)^{-1}}_{\text{Effort component}} \cdot \underbrace{(1-\rho)^{\frac{1}{\theta}}}_{\text{Firm component}} \cdot \underbrace{\left(\frac{\tau_{ij} w_i}{b_i}\right)}_{\text{Country-pair component}}.$$

We introduce efficiency wage using a modified Shapiro-Stiglitz model. Ignoring country subscript for now, let $V_E(\rho)^{Non}$ and $V_E(\rho)^{Shirk}$ be the expected utility of employed non-shirkers and shirkers at firm ρ. Let V_O be the value of an outside option. The fundamental asset equations for employed non-shirkers and shirkers, respectively, are

$$rV_E(\rho)^{Non} = \bar{z}(\rho) - e + \delta\left[V_O - V_E(\rho)^{Non}\right] \tag{B6}$$
$$rV_E(\rho)^{Shirk} = \bar{z}(\rho) + (\delta + u(\rho))\left[V_O - V_E(\rho)^{Non}\right], \tag{B7}$$

with the utility cost of working, e; exogenous job separation rate, δ; and the probability of shirker being fired, $u(\rho)$, which equals the probability of firm ρ detecting shirker. The first line represents the value function of a non-shirker. She earns labor payment $\bar{z}(\rho)$ while incurring the disutilty of working e. She faces an exogenous job separation rate δ where in the event of such job dismissal, she obtains the (negative) net value of unemployment. A shirker, on the contrary, does not incur the disutility of working. Instead, her probability of job separation increases by the probability of detected shirking, $u(\rho)$, which is a function of monitoring technology.

We set the value of the outside option as in the fair wage specification:

$$rV_O = (1-\mu)(1-U)\int_{\rho} \Pi_i(\rho)d\rho,$$

where μ is the profit-sharing parameter derived below and $(1-U)$ is the probability that the individual can be matched with another firm if she leaves the firm ρ. The incentive comparability condition requires that firms pay higher payments to induce the hired individual to prefer to exert effort rather than to shirk and risk getting detected.

To proceed, we need an additional structure for the firm's monitoring technology. If we assume that the monitoring technology, and hence, the probability of detection, is the same across firms, they will offer the same wage. An alternative assumption is that firms draw production productivity and monitoring technology from a copula, as in Davis and Harrigan (2011). In this case, wages depend on monitoring technology but not firm productivity and deriving an analytical expression for the profit sharing function is challenging. Instead, we assume that the monitoring technology is inversely related to the firm's profitability.[12] This may be driven, for instance, by the fact that more profitable firms operate on a larger scale, making it harder for them to detect shirking. For simplicity, we use $u(\rho) = \eta/\Pi(\rho)$, where η is a parameter.

With the above restriction, the efficiency wage function is given as the firm's profits:

$$\bar{z}_i(\rho) = \delta\frac{e}{\eta/\Pi_i(\rho)} + e + rV_{O,i} \tag{B8}$$

Let us use μ to denote $\delta e/\eta$. Then the expression for efficiency wages at firm ρ can be written as:

$$\bar{z}_i(\rho) = \mu\Pi_i(\rho) + w + (1-\mu)(1-U)\int_{\rho} \Pi_i(\rho)d\rho, \tag{B9}$$

where the wage is set to compensate for the disutilities of working, and the third component is given by assumption for the value of the outside option. Note that the expression for efficiency wages is equivalent to the fair wage expression derived in subsection B1. Hence, the rest of the derivations is the same as in subsection B1. We conclude that there is linear profit sharing if the detection technology for shirking is $u(\rho) = \eta/\Pi(\rho)$.

[12] Davis and Harrigan (2011) incorporate the Shapiro-Stiglitz efficiency wage framework into the Melitz (2003) model. They find that a small negative ex-ante correlation between productivity and monitoring ability is needed to simulate a positive size-wage correlation. Our restriction can be viewed as a reduced-form approximation of such an assumption.

B4: Skill assignment model

Next, we consider an assignment between managerial ability and the quality of the idea (i.e., the firm's inherent productivity). We construct a skill assignment model a la Monte (2011) and impose necessary restrictions to attain **R4**. In this subsection, we focus on the implication of assortative matching between firms and managers by considering a model without search-and-matching friction.

As in the baseline model, a firm draws the productivity ϕ from a known country-specific Pareto distribution with the cumulative distribution function $F_i(\phi) = 1 - b_i^\theta \phi^{-\theta}$. We interpret this firm's inherent productivity as the quality of the idea. The idea itself cannot generate any profit unless a manager operates its production activity. We additionally assume that an individual draws her managerial ability ε from a known country-specific Pareto distribution with the cumulative distribution function $G_i(\varepsilon) = 1 - h_i^\psi \varepsilon^{-\psi}$ where $h_i > 0$ is a country-specific scale parameter, and $\psi > 0$ is the shape parameter common to all countries. The total firm productivity of a pair (ϕ, ε) is given as:

$$\varphi(\phi, \varepsilon) = \phi^\kappa \cdot \varepsilon^k, \tag{B10}$$

with productivity weights $\kappa > 0$ and $k > 0$. Equation (B10) highlights the complementarity between the productivity of the idea and the manager. Hence, with profit-sharing, both firms and managers have an incentive to be matched with a more productive partner.

Absent search-and-matching friction, the equilibrium features perfect assortative matching, with $F_i(\phi) = G_i(\varepsilon)$ and $b_i^\theta \phi^{-\theta} = h_i^\psi \varepsilon^{-\psi}$. In other words, a firm at position ρ is matched with a manager at the same position. We impose two restrictions to derive R4: (1) the sum of productivity weights equals one ($\kappa + k = 1$), and (2) the parameters governing distributions of the quality of the idea and the managerial ability are the same ($b_i = h_i$ and $\theta = \psi$). The above restrictions imply that **R4** can be interpreted as a model of perfect assignment between firms and managers.

B5: Monopsony power

In this subsection, we demonstrate that the linear profit sharing mechanism used in the baseline specification can be microfounded using monopsonistic labor markets as in Card et al. (2018). Monopsonistic labor markets have been used in the context of international trade in Egger et al. (2021) and Jha and Rodriguez-Lopez (2021).

As before, to make sure that the conditions R1-R3 are satisfied, we consider inequality stemming from the differential compensation of managers. Consider firms in country i that have firm-specific productivity component ε distributed according to $Pareto(b_i, \theta)$. Let us again rank firms according to the productivity levels and use ρ's to denote their percentile. Individuals have utility function that is log-linear in the managerial income and the amenity level of the firm, $a_i(\rho)$, where she is employed. Individuals also receive random utility shocks for each possible firm match drawn from an extreme value distribution such that the probability that manager ϕ works for firm ρ is as follows:

$$\bar{\phi}_i(\rho) = \frac{z_i(\phi,\rho)^{\mu} a_i(\rho)}{\int_r z_i(\phi,r)^{\mu} a_i(r) dr} = A_i z_i(\phi,\rho)^{\mu} a_i(\rho),$$

where $z_i(\phi,\rho)$ is total managerial compensation offered by firm ρ to the individual ϕ and μ measures the importance of income relative to amenities. Given the probability of hiring manager ϕ, the expected productivity level of the manager is $\varepsilon_i(\rho) = \bar{\phi}_i(\rho)\phi$. Being able to attract managers with better abilities increases firm's overall productivity such that the marginal cost of firm ρ serving market j can be specified as:

$$m_{ij}(\rho) = \underbrace{\varepsilon_i(\rho)^{-1}}_{\text{Manager component}} \cdot \underbrace{(1-\rho)^{\frac{1}{\theta}}}_{\text{Firm component}} \cdot \underbrace{\left(\frac{\tau_{ij} w_i}{b_i}\right)}_{\text{Country-pair component}}.$$

Firm ρ can influence $\varepsilon_i(\rho)$ by offering higher wage to the manager and maximizes total profit net of the payment to the manager conditional on the associated marginal cost function as:

$$\max_{z(\phi,\rho)} \left\{ \sum_j \mathbb{1}_{\rho > \rho^*_{ij}} z_i(\phi,\rho)^{\mu(\sigma-1)} \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \frac{A_i a_i(\rho)}{\phi} (1-\rho)^{\frac{1}{\theta}} \frac{\tau_{ij} w_i}{b_i} \right)^{1-\sigma} P_j^{\sigma-1} L_j y_j - z_i(\phi,\rho) \right\}.$$

In this case, first-order condition with respect to $z_i(\phi,\rho)$ is as follows:[13]

$$z_i^*(\phi,\rho) = (\sigma-1)\mu \left[\sum_j \mathbb{1}_{\rho > \rho^*_{ij}} z_i^*(\phi,\rho)^{\mu(\sigma-1)} \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} A_i a_i(\rho) (1-\rho)^{\frac{1}{\theta}} \frac{\tau_{ij} w_i}{b_i} \right)^{1-\sigma} P_j^{\sigma-1} L_j y_j \right].$$

This means that we can write the managerial income of ϕ as a constant share of profits such that ρ also pins down the income position of ϕ in the income distributions:

$$z_i^*(\rho) = \alpha \Pi_i(\rho), \ \text{ where } \ \alpha = (\sigma-1)\mu \leq 1$$

In this case, total income of individual ρ can be specified as in our baseline equation:

$$y_i(\rho) = w_i + z_i^*(\rho) + d_i = w_i + \alpha \Pi_i(\rho) + (1-\alpha) \int_\rho \Pi_i(\rho) d\rho.$$

Hence, linear profit sharing can be derived in a model based on monopsonistic labor markets where individuals have heterogeneous preferences for firm amenities with random utility shocks drawn from an extreme value distribution.

[13] As before, we make a simplifying assumption that $z_i(\phi,\rho)$ lowers the marginal cost of serving j conditional on entry.

B6: Other settings

There are two limitations of our approach that are worth discussing. First, it potentially ignores the effects that inequality exerts on export selection via firm-specific production costs. Second, the profit-sharing mechanism may not be linear. To examine whether our approach is useful in such settings, we examine a framework based on fair wages with firm-specific *production* wages and multiplicative specification of the fair-wage function. The specification that we consider below is in the spirit of Egger and Kreickemeier (2012).

Let $w_i(\phi)$ denote the wage that firm ϕ pays to workers and let the fair-wage function lead to the following wage setting rule:

$$w_i(\rho) = [\Pi_i(\rho)]^{\upsilon} [(1-U)w_i]^{1-\upsilon} \quad \text{, where } \; w_i = \int_\rho w_i(\rho) d\rho. \tag{B11}$$

As before, U can be interpreted as an exogenous risk of unemployment or costs of moving to another firm and w_i now denotes the average wage. The central difference between this setting and our baseline model is that the marginal cost is now a function of $w_i(\phi)$ such that:

$$m_{ij}(\rho) = \underbrace{(1-\rho)^{\frac{1}{\theta}} w_i(\rho)}_{\text{Firm component}} \cdot \underbrace{\left(\frac{\tau_{ij}}{b_i}\right)}_{\text{Country-pair component}}. \tag{B12}$$

In this case, the marginal firm serving market j is pinned down by the following condition:

$$(1-\rho_{ij}^*)^{\frac{1-\sigma}{\theta}} = w_i \left(\rho_{ij}^*\right)^{\sigma-1} b_i^{\sigma-1} \sigma w_j f_{ij} V_{ij}^{-1}$$

where $V_{ij} = \left(\frac{\sigma}{\sigma-1}\tau_{ij}\right)^{1-\sigma} P_j^{\sigma-1} L_j y_j$ and the profits of firm ρ can be expressed as:

$$\Pi_i(\rho) = \sum_j \mathbb{1}_{\rho > \rho_{ij}^*} \left\{ b_i^{\sigma-1}(1-\rho)^{\frac{1-\sigma}{\theta}} \frac{1}{\sigma} V_{ij} w_i(\rho)^{1-\sigma} - w_j f_{ij} \right\}.$$

Accordingly, we specify the expression for the price index:

$$P_j = \left(\sum_k N_k b_k^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\tau_{kj}\right)^{1-\sigma} \int_{\rho_{kj}^*}^1 (1-\rho)^{\frac{1-\sigma}{\theta}} w_k(\rho)^{1-\sigma} d\rho \right)^{\frac{1}{1-\sigma}},$$

and the expression for trade flows:

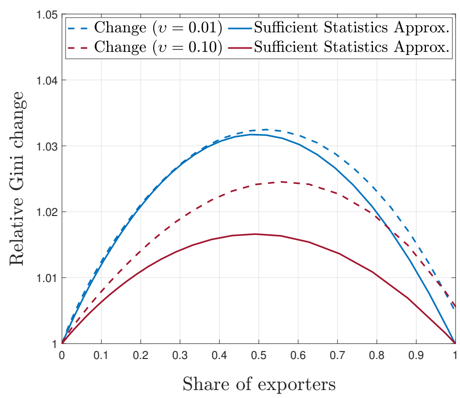
$$X_{ij} = N_i b_i^{\sigma-1} V_{ij} \int_{\rho_{ij}^*}^1 (1-\rho)^{\frac{1-\sigma}{\theta}} w_i(\rho)^{1-\sigma} d\rho,$$

When wages and marginal costs are characterized according to equations (B11) and (B12), total nominal income is comprised of firm-specific wages and any remaining profits as long as they are distributed back to workers. In this case, the expression for nominal income is as follows:

$$y_i(\rho) = w_i(\rho) + \Pi_i(\rho) \text{ s.t. } \int_\rho w_i(\rho)d\rho = w_i. \tag{B13}$$

It turns out that the sufficient statistics formula provides the *lower bound* of the effects of trade on inequality. Note that when $\upsilon = 0$, the expression for nominal income in equation (B13) collapses to its counterpart in equation (1) for $\alpha = 1$. When $\upsilon > 0$ the ACR formula and inequality sufficient statistics approach in Proposition 1 no longer hold. Nevertheless, the proposed sufficient statistics approach is still useful for two reasons. First, it puts the lower bound on the effects of trade on inequality. Second, under plausible values of υ, the approach approximates changes in inequality when profit-sharing is non-linear and there is a feedback between workers' incomes and firms' marginal costs.

To evaluate how closely the sufficient statistics approach approximates predictions of the model with an alternative profit-sharing and cost functions, we return to the results in Figure 1, where we consider the world of I symmetric countries. We solve a version of the model where incomes and marginal costs are generated as in equations (B11), (B12), and (B13) at different values of variable trade costs. At each equilibrium, we record the sufficient statistics $\{X_{ij}, \rho^*_{ij}\}$ as well as the Gini coefficients. We then compare actual Gini coefficients to the predictions of the sufficient statistics approach.



Notes: The figure plots relative changes in the Gini coefficient and their sufficient statistics approximations for different values of υ based on a simulated world economy with $I = 3$ symmetric countries, where $\rho^*_{ii} = 0$. The changes (vertical axis) are calculated using the share of exporters (horizontal axis) in hypothetical equilibria relative to autarky.

Figure B1: Inequality Measures vs. Share of Exporters

We plot relative changes in the Gini coefficient against the share of exporters for different values of υ as dashed curves in Figure B1. We compare them to the predictions based on the sufficient statistics approach that uses $\{X_{ij}, \rho^*_{ij}\}$ plotted as solid curves. When $\upsilon = 0.01$, the sufficient statistics approach very closely approximates actual changes in the Gini coefficients. When $\upsilon = 0.10$, the difference increases but the sufficient statistics approximation still serves as a useful lower bound.

Appendix C: Data Details

For quantitative analysis, we need full matrices of trade flows X_{ij}, country-level domestic operation shares ρ_{ii}^*, and exporter-by-importer exporter shares ρ_{ij}^*. We obtain total trade from the World Input-Output Database (WIOD) (Timmer et al., 2015). The dataset provides country-by-industry level trade flows, which we also use to construct domestic consumption shares. We use the 2016 release, which covers 28 EU countries and 15 other major countries for the period 2000-2014. Those 43 countries and the rest of the world constitute our main sample.

To obtain the domestic operating shares ρ_{ii}^* and the exporter shares ρ_{ij}^*, we compile firm data from INDSTAT (United Nations, 2023), OECD Structural and Demographic Business Statistics (SDBS) (OECD, 2023), OECD Trade by Enterprise Characteristics data (TEC) (OECD, 2021), and World Bank Exporter Dynamics Database (EDD) (World Bank, 2015). INDSTAT is a panel of 175 countries from 1963 onward. It contains information about aggregate firm characteristics of manufacturing firms, such as the number of establishments, employees, total wage, output, and value-added. Similarly, SDSB records detailed information about sector-level data from 1990 onward. We take the number of firms from the two datasets as the denominator for the shares of domestic operating firms and exporters. SDBS also provides the shares of exiting firms which we use to construct the domestic operating shares ρ_{ii}^*.

TEC and EDD record characteristics of exporting sectors, such as the number of exporting firms and export values by firm size, trading partners, and export intensity. TEC covers 66 exporting countries over the 2007-2020 period; EDD covers 67 countries over the 1997-2014 period. We combine the two datasets to maximize sample countries. For our purpose, we use the number of exporting firms at the exporter-importer-year level as the numerator of the exporter shares.

Without any imputations, we collect firm exit rates from 29 countries (67.44%) for the benchmark year of 2014. We also have 1,176 bilateral observations (65.11%) of exporter shares for 2014. The remaining observations are imputed for quantitative analysis. We first separately predict the number of exiting firms (the numerator for ρ_{ii}^*), the number of exporting firms (the numerator for ρ_{ij}^*), and the total number of operating firms (the denominator for both) using a Poisson Pseudo Maximum Likelihood (PPML) regression (Silva and Tenreyro, 2006). Then we calculate the shares.

For the number of total domestic firms and exiting firms, we use three imputation models with different sets of fixed effects to predict missing values. The first model is estimated by log population, squared log population, and log output-side purchasing power parity (PPP) per capita with exporter fixed effects and year fixed effects. The second model drops year-fixed effects, and the third model only uses explanatory variables. Although the first model is our preferred imputation model, we also employ the second and the third models when our preferred model is not feasible. The explanatory variables such as (log) population and (log) PPP per capita are taken from the Penn World Table (Feenstra et al., 2015).

We use a similar approach in imputing the number of bilateral exporters. The first model is estimated by log trade flows with three sets of fixed effects: exporter-year, importer-year, and pair fixed effects. The second model uses only pair fixed effects. The third model uses exporter-, importer-, and year-specific fixed effects.

Appendix D: Sensitivity of the results

In the main text, we set $\theta = 2.5$ and $\sigma = 3$. In this appendix, we report the results for alternative values of both θ and σ. We calculate relative changes in the Gini coefficient for the range of $\theta \in [2.5, 8]$ and $\sigma \in [3, 6]$ (note that changes in the Gini coefficients do not depend on α), while making sure that the constraint $\theta > (\sigma - 1)$ holds. We report counterfactual changes in the Gini coefficient relative to 2014 in Table D1.

country	$\theta = 2.5$ $\sigma = 3$	$\theta = 5$ $\sigma = 3$	$\theta = 8$ $\sigma = 3$	$\theta = 5$ $\sigma = 4$	$\theta = 8$ $\sigma = 4$	$\theta = 5$ $\sigma = 5$	$\theta = 8$ $\sigma = 5$	$\theta = 8$ $\sigma = 6$
AUS	0.97	0.92	0.90	0.94	0.91	0.97	0.93	0.95
AUT	0.95	0.87	0.84	0.91	0.87	0.95	0.89	0.91
BEL	0.94	0.85	0.81	0.89	0.84	0.94	0.87	0.90
BGR	0.94	0.83	0.80	0.88	0.83	0.94	0.86	0.89
BRA	0.97	0.92	0.91	0.95	0.92	0.97	0.93	0.95
CAN	0.96	0.88	0.86	0.92	0.88	0.96	0.90	0.92
CHE	0.97	0.92	0.90	0.94	0.91	0.97	0.93	0.95
CHN	0.98	0.96	0.95	0.97	0.96	0.98	0.96	0.97
CYP	0.94	0.84	0.81	0.89	0.83	0.94	0.86	0.89
CZE	0.93	0.82	0.79	0.87	0.82	0.93	0.85	0.88
DEU	0.96	0.88	0.85	0.92	0.87	0.96	0.90	0.92
DNK	0.95	0.86	0.83	0.90	0.85	0.95	0.88	0.91
ESP	0.96	0.89	0.86	0.92	0.88	0.96	0.90	0.93
EST	0.95	0.86	0.83	0.90	0.85	0.95	0.88	0.91
FIN	0.95	0.87	0.84	0.91	0.86	0.95	0.89	0.91
FRA	0.96	0.89	0.86	0.92	0.88	0.96	0.90	0.93
GBR	0.96	0.88	0.86	0.92	0.88	0.96	0.90	0.92
GRC	0.96	0.88	0.86	0.92	0.88	0.96	0.90	0.92
HRV	0.94	0.84	0.81	0.89	0.84	0.94	0.87	0.90
HUN	0.92	0.80	0.76	0.85	0.79	0.92	0.82	0.86
IDN	0.96	0.90	0.88	0.93	0.89	0.96	0.91	0.93
IND	0.99	0.97	0.96	0.98	0.97	0.99	0.97	0.98
IRL	0.91	0.78	0.74	0.84	0.78	0.91	0.81	0.85
ITA	0.96	0.90	0.88	0.93	0.89	0.96	0.91	0.93
JPN	0.97	0.90	0.88	0.93	0.90	0.97	0.92	0.94
KOR	0.95	0.87	0.84	0.91	0.86	0.95	0.89	0.91
LTU	0.92	0.79	0.75	0.85	0.79	0.92	0.82	0.86
LUX	0.93	0.82	0.78	0.87	0.81	0.93	0.84	0.88
LVA	0.94	0.85	0.82	0.89	0.84	0.94	0.87	0.90
MEX	0.94	0.85	0.82	0.89	0.85	0.94	0.87	0.90
MLT	0.91	0.76	0.72	0.83	0.76	0.91	0.79	0.84
NLD	0.94	0.84	0.80	0.88	0.83	0.94	0.86	0.89
NOR	0.96	0.90	0.87	0.93	0.89	0.96	0.91	0.93
POL	0.95	0.85	0.82	0.90	0.85	0.95	0.87	0.90
PRT	0.95	0.85	0.82	0.90	0.85	0.95	0.88	0.90
ROU	0.95	0.86	0.83	0.90	0.86	0.95	0.88	0.91
ROW	0.96	0.89	0.86	0.92	0.88	0.96	0.90	0.93
RUS	0.96	0.89	0.86	0.92	0.88	0.96	0.90	0.93
SVK	0.93	0.82	0.78	0.87	0.81	0.93	0.84	0.88
SVN	0.94	0.84	0.81	0.89	0.83	0.94	0.86	0.89
SWE	0.96	0.88	0.85	0.92	0.87	0.96	0.90	0.92
TUR	0.95	0.87	0.84	0.91	0.87	0.95	0.89	0.92
TWN	0.94	0.84	0.81	0.89	0.84	0.94	0.86	0.89
USA	0.97	0.93	0.91	0.95	0.92	0.97	0.94	0.95

Notes: The table reports relative changes in the Gini coefficients if each country reverted to autarky relative to 2014 for different values of θ and σ.

Table D1: Relative changes in Gini

We also show how other inequality statistics, i.e., share of the top 5 %, depend on the value of α. For that we set θ and σ as in the baseline calibration and consider different values of α. As before, we report how inequality would change if countries reverted back to autarky relative to 2014 in Table D2.

country	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 0.7$	$\alpha = 0.6$	$\alpha = 0.5$	$\alpha = 0.4$	$\alpha = 0.3$	$\alpha = 0.2$	$\alpha = 0.1$
AUS	0.95	0.95	0.95	0.95	0.96	0.96	0.96	0.97	0.97	0.98
AUT	0.92	0.93	0.93	0.93	0.93	0.94	0.94	0.95	0.96	0.97
BEL	0.91	0.91	0.92	0.92	0.92	0.93	0.93	0.94	0.95	0.97
BGR	0.89	0.89	0.90	0.90	0.90	0.91	0.92	0.93	0.94	0.96
BRA	0.96	0.96	0.96	0.96	0.96	0.96	0.97	0.97	0.98	0.99
CAN	0.93	0.93	0.93	0.93	0.94	0.94	0.94	0.95	0.96	0.98
CHE	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.98	0.99
CHN	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.99	0.99
CYP	0.89	0.89	0.89	0.90	0.90	0.91	0.92	0.93	0.94	0.96
CZE	0.88	0.88	0.89	0.89	0.89	0.90	0.91	0.92	0.93	0.96
DEU	0.93	0.93	0.93	0.93	0.94	0.94	0.95	0.95	0.96	0.98
DNK	0.92	0.93	0.93	0.93	0.93	0.94	0.94	0.95	0.96	0.97
ESP	0.93	0.93	0.94	0.94	0.94	0.94	0.95	0.95	0.96	0.98
EST	0.92	0.93	0.93	0.93	0.93	0.94	0.94	0.95	0.96	0.97
FIN	0.92	0.92	0.92	0.92	0.93	0.93	0.94	0.94	0.96	0.97
FRA	0.93	0.93	0.93	0.93	0.94	0.94	0.94	0.95	0.96	0.98
GBR	0.93	0.93	0.94	0.94	0.94	0.95	0.95	0.96	0.96	0.98
GRC	0.92	0.92	0.93	0.93	0.93	0.94	0.94	0.95	0.96	0.97
HRV	0.90	0.90	0.91	0.91	0.91	0.92	0.93	0.93	0.95	0.97
HUN	0.86	0.87	0.87	0.87	0.88	0.89	0.90	0.91	0.92	0.95
IDN	0.94	0.94	0.94	0.94	0.94	0.95	0.95	0.96	0.97	0.98
IND	0.96	0.96	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.99
IRL	0.86	0.86	0.87	0.87	0.88	0.88	0.89	0.91	0.92	0.95
ITA	0.94	0.94	0.94	0.94	0.95	0.95	0.95	0.96	0.97	0.98
JPN	0.95	0.95	0.95	0.95	0.95	0.96	0.96	0.96	0.97	0.98
KOR	0.92	0.92	0.92	0.93	0.93	0.93	0.94	0.95	0.96	0.97
LTU	0.87	0.87	0.87	0.88	0.88	0.89	0.90	0.91	0.93	0.95
LUX	0.88	0.89	0.89	0.89	0.90	0.90	0.91	0.92	0.94	0.96
LVA	0.90	0.91	0.91	0.91	0.92	0.92	0.93	0.94	0.95	0.97
MEX	0.91	0.92	0.92	0.92	0.93	0.93	0.94	0.94	0.95	0.97
MLT	0.84	0.84	0.85	0.85	0.86	0.86	0.87	0.89	0.91	0.94
NLD	0.89	0.89	0.90	0.90	0.91	0.91	0.92	0.93	0.94	0.96
NOR	0.94	0.94	0.95	0.95	0.95	0.95	0.96	0.96	0.97	0.98
POL	0.91	0.91	0.91	0.92	0.92	0.92	0.93	0.94	0.95	0.97
PRT	0.91	0.91	0.91	0.92	0.92	0.93	0.93	0.94	0.95	0.97
ROU	0.91	0.92	0.92	0.92	0.92	0.93	0.93	0.94	0.95	0.97
ROW	0.92	0.92	0.92	0.93	0.93	0.93	0.94	0.95	0.96	0.97
RUS	0.94	0.94	0.94	0.94	0.95	0.95	0.95	0.96	0.97	0.98
SVK	0.90	0.90	0.90	0.90	0.91	0.91	0.92	0.93	0.94	0.96
SVN	0.90	0.91	0.91	0.91	0.92	0.92	0.93	0.94	0.95	0.97
SWE	0.93	0.93	0.93	0.93	0.94	0.94	0.95	0.95	0.96	0.98
TUR	0.93	0.93	0.93	0.93	0.94	0.94	0.95	0.95	0.96	0.98
TWN	0.89	0.90	0.90	0.90	0.91	0.91	0.92	0.93	0.94	0.96
USA	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.98	0.99

Notes: The table reports relative changes in the share of top 5% if each country reverted to autarky relative to 2014 under different values of α.

Table D2: Relative change in the share of Top 5%